More Trig Identities W.S Solutions.

#1. $\cos x \sec x - \cos^2 x$

$= 1 - \cos^2 x$

$= \boxed{\sec^2 x}$ ✓

#2. $\sin x + \cot x \cdot \cos x$

$= \sin x + \frac{\cos x}{\sin x} \cdot \cos x$

$= \frac{\sin^2 x + \cos^2 x}{\sin x}$

$= \frac{1}{\sin x} = \csc x$

#3. $\frac{\sec^2 x - \tan^2 x + \tan x}{\sec x}$

$= \frac{\left(\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} + \frac{\sin x}{\cos x}\right)\cos^2 x}{\left(\frac{1}{\cos x}\right)\cos^2 x}$

$= \frac{1 - \sin^2 x + \sin x \cos x}{\cos x}$

$= \frac{\cos^2 x + \sin x \cos x}{\cos x}$

$= \frac{\cos x(\cos x + \sin x)}{\cos x}$

$= \boxed{\cos x + \sin x}$

#4. $\frac{\sin x}{1 + \cos x} + \cot x$

$= \frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos x(1 + \cos x)}{\sin x(1 + \cos x)}$

$= \frac{(1 + \cos x)}{\sin x(1 + \cos x)} = \frac{1}{\sin x}$

$= \boxed{\csc x}$

#5. $1 - \dfrac{\sin^2 x}{1-\cos x}$

$= \dfrac{(1-\cos x) - \sin^2 x}{1-\cos x}$

$= \dfrac{(1-\sin^2 x) - \cos x}{1-\cos x}$ ($1-\sin^2 x = \cos^2 x$)

$= \dfrac{\cos^2 x - \cos x}{1-\cos x}$

$= \dfrac{\cos x(\cos x - 1)}{(1-\cos x)}$ ($\cos x - 1 = -(1-\cos x)$)

$= \boxed{-\cos x}$

#6 $\dfrac{1+\csc x}{\cot x + \cos x}$

$= \dfrac{\left(1+\dfrac{1}{\sin x}\right)\sin x}{\left(\dfrac{\cos x}{\sin x} + \cos x\right)\sin x}$

$= \dfrac{\sin x + 1}{\cos x + \cos x \cdot \sin x}$

$= \dfrac{\sin x + 1}{\cos x(1+\sin x)}$

#7. $\dfrac{1+\sin x - \sin^2 x}{\cos x} = \dfrac{1-\sin^2 x + \sin x}{\cos x} = \dfrac{\cos^2 x}{\cos x} + \dfrac{\sin x}{\cos x}$

$= \boxed{\cos x + \tan x}$

#8. $(\csc x + \cot x)(1-\cos x) = \csc x - \csc x \cdot \cos x + \cot x - \cot x \cos x$

$= \csc x - \dfrac{\cos x}{\sin x} + \cot x - \dfrac{\cos x}{\sin x} \cdot \cos x$

$= \dfrac{1}{\sin x} - \cot x + \cot x - \dfrac{\cos^2 x}{\sin x} = \dfrac{1-\cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \boxed{\sin x}$

#9. $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x}$

$= \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} + \frac{\sin x}{\cos x}$

$= \frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}$

$= \boxed{2\tan x}$

#10 $\tan x + \frac{\cos x}{1+\sin x}$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}$

common denominator

$= \frac{\sin x(1+\sin x) + \cos x \cdot \cos x}{(\cos x)(1+\sin x)}$

$= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)}$ "1"

$= \frac{\sin x + 1}{\cos x(1+\sin x)} = \frac{1}{\cos x} = \boxed{\sec x}$

#11 $\frac{\sec x + \tan x}{\cot x + \cos x}$

$= \frac{\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) \cdot \cos x \cdot \sin x}{\left(\frac{\cos x}{\sin x} + \cos x\right) \cdot \cos x \cdot \sin x}$

$= \frac{\sin x + \sin^2 x}{\cos^2 x + \cos^2 x \sin x} =$

$= \frac{\sin x(1+\sin x)}{\cos^2 x(1+\sin x)} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \boxed{\tan x \cdot \sec x}$

#12. $\sec x - \cos x - \sin x \tan x$

$= \left(\frac{1}{\cos x}\right) - \cos x - \left(\sin x \cdot \frac{\sin x}{\cos x}\right)$

$= \frac{1 - \sin^2 x}{\cos x} - \cos x$

$= \frac{\cos^2 x}{\cos x} - \cos x$

$= \cos x - \cos x = \boxed{1}$