Pre HL Math Induction Exit Slip

Name: Key

No Calculator

1. By mathematical induction, prove that $\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ for all integers n, $n \geq 1$.

1) If $n=1 \Rightarrow (1)(1+2)=3 \Rightarrow \frac{(1)(1+1)(2+7)}{6} = 3$ The conjecture $\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ is true.

2) If $n=k \Rightarrow$ Assume $\sum_{i=1}^{k} i(i+2) = \frac{k(k+1)(2k+7)}{6}$

is true for $n=k$ $k \in \mathbb{Z}^+$

3) If $n=k+1 \Rightarrow \sum_{i=1}^{k} i(i+2) + (k+1)(k+1+2)$

$= \frac{k(k+1)(2k+7)}{6} + \frac{6(k+1)(k+3)}{6} = \frac{(k+1)}{6}[k(2k+7)+(6k+18)]$

$= \frac{(k+1)}{6}[2k^2+7k+6k+18] = \frac{k+1}{6}[2k^2+13k+18]$

$= \frac{k+1}{6}[(k+2)(2k+9)] = \frac{(k+1)(k+2)(2(k+1)+7)}{6}$

The conjecture is true for $k+1$

$\therefore \sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ is true for Integers $n \geq 1$.

2. Prove, by mathematical induction, that $3^n - 1 - 2n$, $n \in \mathbb{Z}^+$, is divisible by 4.

1) If $n=1 \Rightarrow 3^1 - 1 - 2(1) = 0$

0 is divisible by 4.

2) If $n=k$. $k \in \mathbb{Z}^+$ Assume $\boxed{3^k - 1 - 2k = 4 \cdot A}$ works for $k \in \mathbb{Z}^+$ $\Rightarrow$

Rewrite $3^k = 4 \cdot A + 1 + 2k$

3) If $n=k+1 \Rightarrow 3^{k+1} - 1 - 2(k+1)$

$= 3[(4 \cdot A + 1 + 2k)] - 1 - 2k - 2$

$= 3[4A + 1 + 2k] - 1 - 2k - 2 = 12A + \cancel{3} + 6k - \cancel{1} - 2k - \cancel{2}$

$= 12A + 4k = 4[3A + k]$ which is divisible by 4

4) $\therefore$

3. $\sum_{i=1}^{n}(2i+1)2^{i-1} = 1+(2n-1)\times 2^n$ for all positive integers n.

1) If $n=1 \Rightarrow ((2)(1)+1)\cdot 2^{1-1} = 3$

$\Rightarrow 1+(2\cdot 1-1)\cdot 2^1 = 3$. The Formula is true for $n=1$

2) If $n=k$ where $k \in Z^+$ $\Rightarrow$ Assume $\sum_{i=1}^{k}(2i+1)\cdot 2^{i-1} = 1+(2k-1)\times 2^k$ is true

3) If $n=k+1 \Rightarrow \sum_{i=1}^{k}(2i+1)\cdot 2^{i-1} + (2(k+1)+1)\cdot 2^{k+1-1}$

$= 1+(2k-1)\boxed{2^k} + (2(k+1)+1)\boxed{2^k}$

$= 1+2^k[2k-1+2k+3]$

$= 1+2^k[4k+2]$

$= 1+2^k\cdot 2[2k+1]$

$= 1+2^{k+1}[2(k+1)-1]$.

$\therefore$ The formula $\sum_{i=1}^{n}(2i+1)\cdot 2^{i-1} = 1+(2n-1)\times 2^n$ is true for $n \in Z^+$.