

IB Pre HL

Intro to Rational Functions

For each function, graph it on your GC to find the values where there is a vertical asymptote and/or a hole.

Name: \_\_\_\_\_

Period: \_\_\_\_\_

key

Function	Function in Factored Form	x-values that make the denominator equal 0	Vertical asymptotes	Holes (x, y)	x-intercepts when y=0	y-intercepts when x=0	Domain
1. $f(x) = \frac{x+3}{x^2-9}$	$= \frac{\cancel{x+3}}{(x+3)(x-3)} = \frac{1}{x-3}$	$x=3$ $x=-3$	$x=3$	$x = -3$ $y = \frac{1}{-3-3} = -\frac{1}{6}$	$0 = \frac{1}{x-3}$ None	$y = \frac{1}{0-3} = -\frac{1}{3}$ $(0, -\frac{1}{3})$	$\mathbb{R}$ $x \neq \pm 3$
2. $f(x) = \frac{x-3}{x^2-9}$	$= \frac{\cancel{x-3}}{(x+3)(x-3)} = \frac{1}{x+3}$	$x=3$ $x=-3$	$x=-3$	$x=3$ $y = \frac{1}{6}$	None	$(0, \frac{1}{3})$	$\mathbb{R}$ $x \neq \pm 3$
3. $f(x) = \frac{3}{x^2-9}$	$= \frac{3}{(x+3)(x-3)}$	$x=3$ $x=-3$	$x=3, x=-3$	None	$0 = \frac{3}{x^2-9}$ None	$y = \frac{3}{-9} = -\frac{1}{3}$ $(0, -\frac{1}{3})$	$\mathbb{R}$ $x \neq \pm 3$
4. $f(x) = \frac{9}{x^2+9}$	$= \frac{9}{x^2+9}$	None	None	None	$0 = \frac{9}{x^2+9}$ None	$y = \frac{9}{0+9} = 1$ $(0, 1)$	$\mathbb{R}$
5. $f(x) = \frac{x+2}{x^2-4}$	$= \frac{(x+2)}{(x+2)(x-2)} = \frac{1}{x-2}$	$x=2$ $x=-2$	$x=2$	$x=-2$ $y = \frac{1}{-2-2} = -\frac{1}{4}$	None	$(0, -\frac{1}{2})$	$\mathbb{R}$ $x \neq \pm 2$
6. $f(x) = \frac{x-2}{x^2-4}$	$= \frac{(x-2)}{(x+2)(x-2)} = \frac{1}{x+2}$	$x=2$ $x=-2$	$x=-2$	$x=2$ $y = \frac{1}{4}$	None	$(0, \frac{1}{2})$	$\mathbb{R}$ $x \neq \pm 2$
7. $f(x) = \frac{x-1}{x^2+2x-3}$	$= \frac{(x-1)}{(x+3)(x-1)} = \frac{1}{x+3}$	$x=-3$ $x=1$	$x=-3$	$x=1$ $y = \frac{1}{1+3} = \frac{1}{4}$	None	$(0, \frac{1}{3})$	$\mathbb{R}$ $x \neq -3$
8. $f(x) = \frac{x+3}{x^2+2x-3}$	$= \frac{(x+3)}{(x+3)(x-1)} = \frac{1}{x-1}$	$x=-3$ $x=1$	$x=1$	$x=-3$ $y = -\frac{1}{4}$	None	$(0, -1)$	$\mathbb{R}$ $x \neq 1$
9. $f(x) = \frac{x+1}{x^2+3x+2}$	$= \frac{(x+1)}{(x+1)(x+2)} = \frac{1}{x+2}$	$x=-1$ $x=-2$	$x=-2$	$x=-1$ $y = 1$	None	$(0, \frac{1}{2})$	$\mathbb{R}$ $x \neq -2$
10. $f(x) = \frac{x+2}{x^2-x-6}$	$= \frac{(x+2)}{(x+2)(x-3)} = \frac{1}{x-3}$	$x=-2$ $x=3$	$x=3$	$x=-2$ $y = -\frac{1}{5}$	None	$(0, -\frac{1}{3})$	$\mathbb{R}$ $x \neq 3$
11. $f(x) = \frac{x^2+6x+9}{x^2-9}$	$= \frac{(x+3)^2}{(x+3)(x-3)} = \frac{x+3}{x-3}$	$x=3$ $x=-3$	$x=3$	$x=-3$ $y = 0$	$0 = \frac{x+3}{x-3}$ $x=-3$	$(0, -1)$	$\mathbb{R}$ $x \neq \pm 3$
12. $f(x) = \frac{x^2-1}{2x^3-x^2-2x+1}$	$= \frac{(x+1)(x-1)}{(x+1)(x-1)(2x-1)} = \frac{1}{2x-1}$	$x=1$ $x=-1$ $x=\frac{1}{2}$	$x=\frac{1}{2}$	$x=1$ $y = 1$ $x=-1$ $y = -\frac{1}{3}$	None	$(0, -1)$	$\mathbb{R}$ $x \neq \pm \frac{1}{2}$

$2x^3 - x^2 - 2x + 1 = (2x-1)(x^2 - 1) = (2x-1)(x-1)(x+1)$