

$$\arctan x = \tan^{-1}(x) \neq \frac{1}{\tan x} = \cot x$$

What is inverse trigonometric?

If $\tan \theta = \frac{1}{\sqrt{3}}$ is given, θ is found by $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$. In order not to be confused by the inverse trig $\tan^{-1} x$

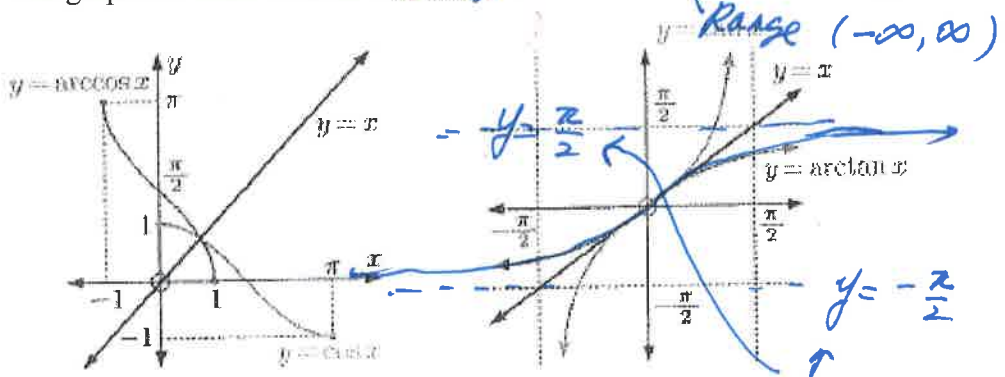
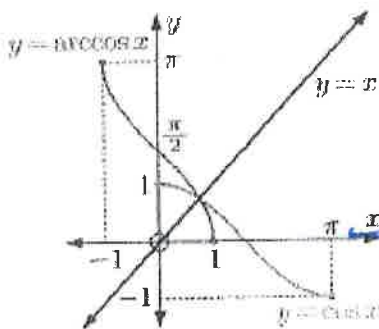
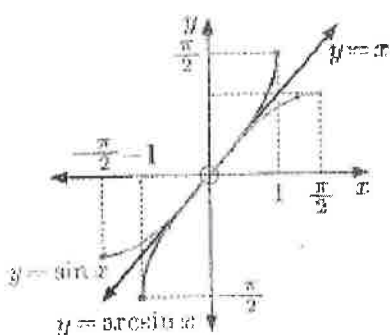
and the reciprocal trig $\frac{1}{\tan x}$, $\tan^{-1} x$ is also expressed $\arctan x$.

Since $\sin x$, $\cos x$, and $\tan x$ are all many to one functions, their domains must be restricted in order for them to have inverse functions.

$$y = \sin x \quad \text{Range: } [-1, 1] \quad \text{Domain } (-\infty, \infty)$$

Functions	Invers functions	Definition and domain	Range
$y = \sin x \Rightarrow x = \sin y$	$y = \arcsin x$ ($y = \sin^{-1} x$)	$x = \sin y \quad -1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos x$	$y = \arccos x$ ($y = \cos^{-1} x$)	$x = \cos y \quad -1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan x$	$y = \arctan x$ ($y = \tan^{-1} x$)	$x = \tan y \quad x \in \mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

The graphs of these inverse functions.

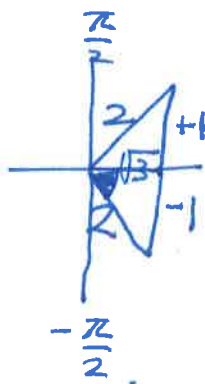


Example 1) Find the exact value of $\arcsin\left(-\frac{1}{2}\right)$.

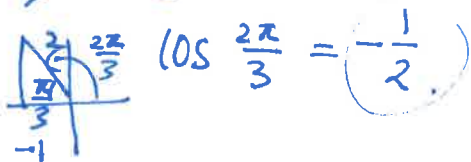
$$-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$$

$$x = -30^\circ$$

$$\text{OR } -\frac{\pi}{6}$$



Example 2) Find the exact value of $\arctan\left(\cos\frac{2\pi}{3}\right) \Rightarrow \arctan\left(-\frac{1}{2}\right) = ?$

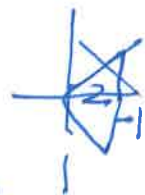


$$-\frac{\pi}{2} \leq \arctan x \leq \frac{\pi}{2}$$

$$-0.46$$

Must use

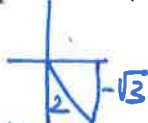
a calculator.



Practice)

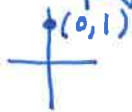
Find the exact value of each expression, if it exists.

$$5. \arcsin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = \boxed{-\frac{\pi}{3}}$$



$$6. \cos^{-1} \left(\cos \frac{\pi}{3} \right) = \boxed{\frac{\pi}{3}}$$

$$7. \tan \left(-\frac{3\pi}{2} \right) = \frac{1}{0} = \boxed{\text{und.}}$$

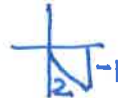


$$8. \sin^{-1} \left(\cos \frac{\pi}{3} \right) = \sin^{-1} \left(\frac{1}{2} \right) = \boxed{\frac{\pi}{6}}$$

$$9. \arctan \left(-\frac{\sqrt{3}}{3} \right) = \arctan \left(-\frac{1}{\sqrt{3}} \right) = \boxed{-\frac{\pi}{6}}$$



$$10. \arcsin \left(-\frac{1}{2} \right) = \boxed{-\frac{\pi}{6}}$$

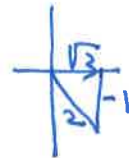


$$11. \tan \left(\sin^{-1} 1 - \cos^{-1} \frac{1}{2} \right)$$

$$12. \sin \left(\arctan -\frac{\sqrt{3}}{3} \right) = \sin \arctan -\frac{1}{\sqrt{3}}$$

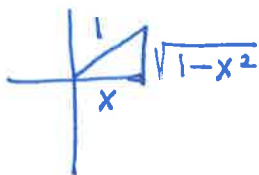
$$\tan \left(\frac{\pi}{2} - \frac{\pi}{3} \right) = \tan \left(\frac{3\pi}{6} - \frac{2\pi}{6} \right) = \tan \left(\frac{\pi}{6} \right) = \boxed{\frac{1}{\sqrt{3}}}$$

$$= \boxed{-\frac{\pi}{6}}$$



Write each trigonometric expression as an algebraic expression of x .

$$14. \sin (\arccos x) = \boxed{\sqrt{1-x^2}}$$



$$15. \tan (\sin^{-1} x) = \boxed{\frac{x}{\sqrt{1-x^2}}}$$

