#1. $\tan 2\theta = \tan(\theta+\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{3}{4}.$

①

$$8\tan\theta = 3(1-\tan^2\theta)$$

$$8\tan\theta = 3 - 3\tan^2\theta$$

$$+3\tan^2\theta - 3 \quad -3 \quad +3\tan^2\theta$$

$$3\tan^2 + 8\tan\theta - 3 = 0$$

$$3\tan\theta \qquad -1$$

$$\tan\theta \qquad +3$$

$$\Rightarrow (3\tan\theta - 1)(\tan\theta + 3) = 0$$

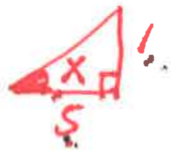
$\tan\theta = \frac{1}{3}$

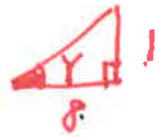
$\tan\theta = -3$

2

#2 (a) $\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \arctan\left(\frac{1}{p}\right)$

$\quad\ \ /\!/ \, x \qquad\qquad /\!/ \, Y \qquad\qquad /\!/ \, z$





Tan $\quad$ tan

$(x + y) = (z)$

$\tan(x+y) = \tan(z)$

$$= \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5}\cdot\frac{1}{8}} = \frac{\frac{13}{40}}{\frac{40}{40} - \frac{1}{40}}$$

$$= \frac{\frac{13}{40}}{\frac{39}{40}} = \frac{13}{39} = \frac{1}{3}.$$

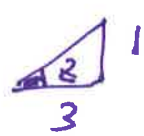
$\arctan\left(\frac{1}{3}\right) = z \qquad \boxed{p = 3}$

(b) $\arctan\left(\frac{1}{2}\right) + \underbrace{\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)}_{\arctan\left(\frac{1}{3}\right) \text{ from (a)}}$

$= \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right)$

$\qquad /\!/ \qquad\qquad /\!/$

$\qquad w \qquad\qquad z$



$\Rightarrow \quad w + z$

$$\Rightarrow \quad \tan(w+z) = \frac{\tan w + \tan z}{1 - \tan w \cdot \tan z} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}}.$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\arctan(1) = \frac{\pi}{4}$

$\arctan(-1) = -\frac{\pi}{4}$

$$\tan(w+z) = 1 \Rightarrow w+z = \arctan(1) = \boxed{\frac{\pi}{4}}$$

$$-\frac{\pi}{2} \leq \arctan(x) \leq \frac{\pi}{2}$$

IB Types of Questions for your Practice

1 Given that $\tan 2\theta = \frac{3}{4}$, find the possible values of $\tan\theta$.

2

(a) Given that $\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \arctan\left(\frac{1}{p}\right)$, where $p \in \mathbb{Z}^+$, find p.

(b) Hence find the value of $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)$.

3

Solve the following for $0 \le x \le 2\pi$.

a. $2 - \sin x = 2\cos^2 x$ b. $\sqrt{3}\cos x + \sin x = 0$

c. $3\sin x - \cos 2x = 1$ d. $\sin 2x + \sin x = 0$

e. $\sqrt{3}\sin x + \cos x = 0$ f. $\sin x - \sqrt{2}\cos x = \frac{1}{2}\sqrt{3}$

#3. (a) $2 - \underline{\sin x} = 2\underline{\cos^2 x}$.

$\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2 - \sin x = 2(1 - \sin^2 x)$

$\cancel{2} - \sin x = \cancel{2} - 2\sin^2 x$.

$+2\sin^2 x \quad +2\sin^2 x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$[0, 2\pi]$

① $\sin x = 0$

② $\sin x = \frac{1}{2}$.

$\boxed{x = 0, 2\pi, \pi}$.

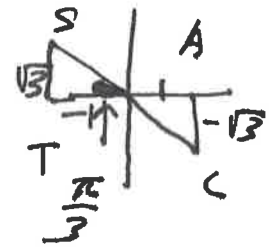
$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

(b) $\dfrac{\sqrt{3}\cos x + \sin x}{\cos x} = \dfrac{0}{\cos x}$ $\tan x = \dfrac{\sin x}{\cos x}$.

$\cos x \neq 0$.

$\Rightarrow \sqrt{3} + \tan x = 0$

$\Rightarrow \tan x = -\sqrt{3}$ $[0, 2\pi]$



$x = \dfrac{2\pi}{3}, \dfrac{5\pi}{3}$

(c) $3\sin x - \cos 2x = 1$ $\cos 2x = 1 - 2\sin^2 x$

$3\sin x - (1 - 2\sin^2 x) = 1$

$3\sin x - 1 + 2\sin^2 x - 1 = 0$

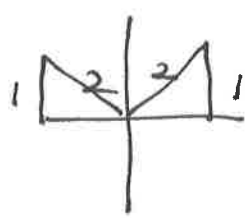
$2\sin^2 x + 3\sin x - 2 = 0$

$2\sin x \quad -1$

$\sin x \quad +2$

$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ $\cancel{\sin x = -2}$



$\boxed{x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}}$

(d) $\sin 2x + \sin x = 0$

$2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$

$\sin x = 0 \qquad \cos x = -\frac{1}{2}.$ $\qquad [0, 2\pi]$

$x = 0, \pi, 2\pi \qquad x = \frac{2\pi}{3}, \frac{4\pi}{3}.$

e. $\frac{\sqrt{3}\sin x + \cos x}{\sin x} = \frac{0}{\sin x}$

$\sqrt{3} + \cot x = 0$

$\cot x = -\sqrt{3}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Name: ______________ Group Members: ______________________

Exploration 5-2a: Linear Combination of Cosine and Sine

Date: __________

Objective: Write the linear combination $y = b\cos\theta + c\sin\theta$ as $y = A\cos(\theta - D)$, a sinusoid with a phase displacement.

The expression on the right in the equation

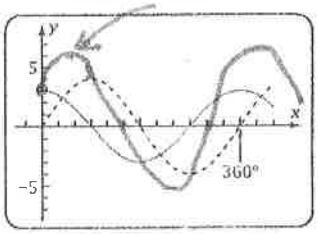
$y = 3\cos\theta + 4\sin\theta$

is called a linear combination of $\cos\theta$ and $\sin\theta$. That is, y equals a constant times cosine, plus a constant times sine. In this Exploration, you will learn how to express such a linear combination as a cosine with a phase displacement.

1. The graph shows

   $y_1 = 3\cos$ and $y_2 = 4\sin\theta$

   Which graph is which?



2. Plot y_3 and sketch it on the figure.

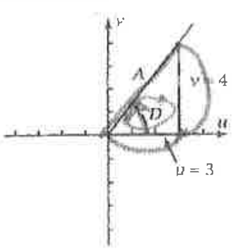
   $y_3 = 3\cos\theta + 4\sin\theta$

3. The graph of y_3 is a sinusoid. Find the amplitude A and the phase displacement D using the MAXIMUM feature of your grapher.

   $A = \sqrt{3^2+4^2} = 5$ $D =$ 53.1

4. Plot $y_4 = A\cos(\theta - D)$ using Problem 3 results. Does the graph coincide with y_3? $5\cos(\theta - 53.1^\circ)$

5. The uv-diagram here shows an angle with $u = 3$, the coefficient of cosine in y_3, and $v = 4$, the coefficient of sine. Show that the hypotenuse equals A from Problem 3.



$y = \underset{X}{3\cos\theta} + \underset{Y}{4\sin\theta}$

$A = \sqrt{3^2+4^2} = 5$

6. Show that the angle D in Problem 3 is a value of $\arctan\frac{4}{3}$, as shown in the figure in Problem 5.

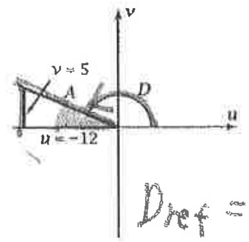
$D = \arctan\left(\frac{4}{3}\right)$

$\approx 53.1^\circ$

7. Express as a cosine with a phase displacement:

   $y = -12\cos\theta + 5\sin\theta$

   Use the next uv-diagram to find the amplitude A and the phase displacement D. Show that D is a value of $\arctan\frac{5}{-12}$ but not the value of $\tan^{-1}\frac{5}{-12}$ that your calculator gives you.



$A = \sqrt{12^2+5^2}$

$= 13$

144
25
169

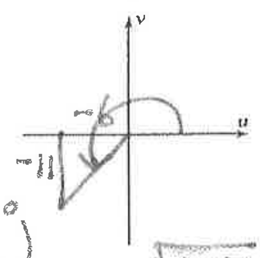
$D_{ref} = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.6^\circ$

$y =$ $D = 180 - 22.6^\circ = 157.4$

$y = 13\cos(\theta - 157.4)$

8. Express as a cosine with a phase displacement:

   $y = -6\cos\theta - 11\sin\theta$



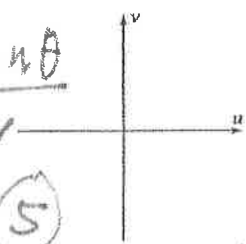
$\cos$ $\tan^{-1}\left(\frac{11}{6}\right) \approx 61$

$y =$ $\sqrt{157}\cos(X - \cancel{74}.4^\circ)$

241.4°

9. Express as a cosine with a phase displacement:

   $y = 9\cos\theta - 7\sin\theta$



$y =$ $y = \sqrt{130}\cos(X + 37.9^\circ)$

$\sqrt{130}\cos(X - 322.1^\circ)$

$\Rightarrow A\cos(\theta - D)$

$A = 5$ $D = 53.1$