IB Pre HL **Introduction to roof:**

Name ________________
Period _____ Date _______

With your team members, discuss what deductive reasoning proof and inductive reasoning proof are. Summarize your discussion. Come up with an example for each if you could.

1) Deductive Reasoning is:

proof process to reason with mathematical formula and Algebraic steps.

2) Inductive Reasoning is:

Making a conjecture based on specific observation and patterns.

Mathematical Induction: Proof of a conjecture.

Conjecture is a statement that is believed to be true (based on the observation of the patterns or induction) but not yet proved.

Math Induction proof process for a given conjecture (statement):

Step 1: Show that the statement is true for n=1.
Step 2: Assume that the statement is true for n=k where $k \in Z^+$.
Step 3: Prove that the statement is true for n=k+1.
Step 4: $\therefore$ The statement is true for $n \in Z^+$

Example)

Given a series: $S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} \ldots\ldots + \frac{1}{n(n+1)}$

$S_1 = \frac{1}{1\times 2} = \frac{1}{2}$

$S_2 = \frac{1}{1\times 2} + \frac{1}{2\times 3} = \frac{2}{3}$

$S_3 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} = \frac{3}{4}$

$S_4 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} = \frac{4}{5}$

$S_5 = \frac{5}{6}$

Make a Conjecture: $S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} \ldots\ldots + \frac{1}{n(n+1)} = \; ?\; \boxed{\frac{n}{n+1}}$ for $n \in Z^+$, key

Prove the above conjecture using Math Induction.

Conjecture: $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$ for $n \in Z^+$

1) When $n=1 \Rightarrow \frac{1}{1\times2} = \frac{1}{2} = \frac{1}{1+1} = \frac{1}{2}$. (The conjecture is true for $n=1$)

2) Assume the conjecture is true for $n=k$ ($k \in Z^+$).

$$\frac{1}{1\times2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} \cdots \frac{1}{k(k+1)} = \frac{k}{k+1}.$$

3) If $n=k+1$. $\Rightarrow$ $\underbrace{\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} \cdots \frac{1}{k(k+1)}}_{= \frac{k}{k+1} \text{ (from step 2)}} + \frac{1}{(k+1)(k+2)}$

$\uparrow$ $n=k+1$ term.

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{(k)(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)}$$

$$= \boxed{\frac{k+1}{k+2}}$$

$\therefore$ Conjecture is true by Induction for $n \in Z^+$.

Practice) Conjecture: $S_n = 1+5+9 \ldots\ldots (4n-3) = n(2n-1)$.

Using Math induction, prove this statement is true for $n \in Z^+$.

1) When $n=1$.

$1 = (1)(2\cdot n - 1) = 1\cdot1 = 1$

(The conjecture is true for $n=1$).

2) Assume the conjecture is true for $n=k$ $\Rightarrow$ $1+5+9 \cdots (4k-3) = k(2k-1)$

3) Show (If $n=k+1$).

Step 2.

$$\underbrace{1+5+9 \cdots (4k-3)}_{n=k} + \underbrace{(4(k+1)-3)}_{n=k+1} = k(2k-1) + (4(k+1)-3)$$

$$= 2k^2 - k + 4k + 4 - 3$$

$$= 2k^2 + 3k + 1 = (k+1)(2k+1)$$

$$= (k+1)(2(k+1)-1)$$

$\therefore$ The conjecture is true for $n \in Z^+$