IB Pre HL **Introduction to roof:** (Day two)

Name key
Period ______ Date ______

Mathematical Induction: Proof of a conjecture. Notes.

Conjecture is a statement that is believed to be true (based on the observation of the patterns or induction) but not yet proved.

Math Induction proof process for a given conjecture (statement):

Step 1: Show that the statement is true for n=1.
Step 2: Assume that the statement is true for n=k where $k \in Z^+$.
Step 3: Prove that the statement is true for n=k+1.
Step 4: $\therefore$ The statement is true for $n \in Z^+$

Example 1) Prove a Conjecture: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for $n \in Z^+$,

1) When $n=1 \Rightarrow (1)^2 = \frac{(1)(1+1)(2\cdot 1+1)}{6} = \frac{6}{6} = 1$

The conjecture is true for $n=1$

2) Assume the conjecture is true for $n=k$ where $k \in Z^+$

$1^2+2^2+3^2 \cdots k^2 = \frac{k(k+1)(2k+1)}{6}$

From the assumption

3) When $n=k+1$.

$1^2+2^2+3^2 \cdots k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{(k+1)[k(2k+1)+6(k+1)]}{6}$

$= \frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} =$

$= \frac{(k+1)(k+2)[2(k+1)+1]}{6}$.

4) $\therefore$ The conjecture is true for $n \in Z^+$.

Example 2) Conjecture: $\sum_{i=1}^{n}(2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

Using Math induction, prove this statement is true for $n \in Z^+$.

1) When $n=1$ $\Rightarrow$ $(2(1)-1)^2 = 1 = \frac{(1)(2\cdot 1-1)(2\cdot 1+1)}{3} = 1$.

The conjecture is true for $n=1$.

2) Assume the conjecture is true for $n=k$ when $k \in Z^+$

$$\Rightarrow \sum_{i=1}^{n}(2i-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

3) When $n=k+1$

$$\Rightarrow \sum_{i=1}^{n}(2i-1)^2 + (2(k+1)-1)^2$$

From step 2.

$$= \frac{k(2k-1)(2k+1)}{3} + (2(k+1)-1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$2k^2+5k+3$

$2k \quad +3$

$k \quad +1$

$$= \frac{(2k+1)[k(2k-1)+3(2k+1)]}{3}$$

$$= \frac{(2k+1)[2k^2-k+6k+3]}{3} = \frac{(2k+1)[2k^2+5k+3]}{3}$$

$$= \frac{(2k+1)(2k+3)(k+1)}{3} = \frac{(k+1)[2(k+1)-1][2(k+1)+1]}{3}$$

4) $\therefore$ The conjecture is true for $n \in Z^+$

Practice) Conjecture: $S_n = \frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + ... + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$.

Using Math induction, prove this statement is true for $n \in Z^+$.

1) When $n=1$ $\Rightarrow \frac{1}{1\times 3} = \frac{1}{2\cdot 1+1} = \frac{1}{3}$

The conjecture is true for $n=1$

2) Assume the conjecture is true for $n=k$ where $k \in Z^+$

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}.$$

Step 2

3) When $n = k+1$

$$\underbrace{\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots \frac{1}{(2k-1)(2k+1)}} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3)}{(2k+1)(2k+3)} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{(2(k+1)+1)}$$

4) $\therefore$ The conjecture is true for $n \in Z^+$.