key

IB Pre HL More Complex number Practice

Example 1) Given that $z_1 = 5+5i$, $z_2 = 1+2i$ and $z_3 = 3-2i$, calculate:

a. $\dfrac{z_1}{z_2}$

$$= \frac{5+5i}{1+2i} \cdot \frac{(1-2i)}{(1-2i)}$$

$$= \frac{5+5i-10i-10i^2}{1^2+2^2}$$

$$= \frac{15-5i}{5} = \boxed{3-i}$$

b. $\dfrac{z_1^{\,2}}{z_2 \cdot z_3^{*}}$

$$= \frac{(5+5i)^2}{(1+2i)\cdot(3+2i)}$$

$$= \frac{25+50i+25i^2}{3+2i+6i+4i^2}$$

$$= \frac{50i}{-1+8i} \cdot \frac{(-1-8i)}{(-1-8i)}$$

$$= \frac{50(-i-8i^2)}{(-1)^2+8^2} = \frac{50(8-i)}{65} = \frac{10(8-i)}{13}$$

$$= \boxed{\frac{10}{13} - \frac{10}{13}i}$$

Example 2) Find the complex number z that satisfies $\dfrac{z+1}{3+i} = \dfrac{z-5i}{2i-1}$

$$\Rightarrow (z+1)(2i-1) = (z-5i)(3+i)$$

$$\Rightarrow z(2i-1) + (2i-1) = z(3+i) - 5i(3+i)$$

$$\Rightarrow z(2i-1) - z(3+i) = -2i+1-15i+5$$

$$\Rightarrow z(-4+i) = 6-17i$$

$$\Rightarrow z = \frac{(6-17i)(-4-i)}{(-4+i)\cdot(-4-i)}$$

$$\Rightarrow z = \frac{-24+68i-6i-17}{16+1} = \boxed{\frac{-41+62i}{17}}$$

More Practice Worksheet

1 Calculate:

a $i^5 + i^8 + i^{14} + i^{19}$

b $i^{123} + i^{172} + i^{256} + i^{375}$

c $(2 - i^{53}) \cdot (3 + 2i^{89})$

d $\dfrac{4i^{2010} - 3i^{2011}}{2i^{2012} + 5i^{2013}}$

e $\dfrac{i + i^2 + i^3 + \ldots + i^{2011}}{i \cdot i^2 \cdot i^3 \cdot \ldots \cdot i^{2011}}$

f $\dfrac{i^2 + i^4 + i^6 + \ldots + i^{2010}}{i^2 \cdot i^4 \cdot i^6 \cdot \ldots \cdot i^{2010}}$

1 Given that $z_1 = 1 + 4i$, $z_2 = 2 - i$, $z_3 = \frac{1}{2} - \frac{5}{2}i$ and $z_4 = \frac{2i - 1}{3}$,

Calculate these quotients and check your answers with a GDC.

a $\dfrac{z_1}{z_2}$ **b** $\dfrac{z_1^*}{z_4}$ **c** $\dfrac{z_2 \cdot z_4}{z_3}$ **d** $\dfrac{3z_1 - 2z_3}{z_2 + 3z_4}$ **e** $\dfrac{z_1^2}{(z_2^*)^2}$

2 Find the real numbers a and b that satisfy these equations.

a $(2 + i)(a + ib) = 11 - 2i$

b $\dfrac{a + ib}{2 - 5i} = -3 + 2i$

c $(3i - 2)(a + ib) = 3 + 28i$

d $\left(\frac{1}{2} + \frac{3}{4}i\right)(a + ib) = -3 + 2i$

5 Find the complex number z that satisfies these equations.

a $(z + 1)i = (z + 2i)(3 + 2i)$

b $(2z - 1)(1 + i) = (z - 1)(2 + 3i)$

c $\dfrac{z - 3i + 2}{4 + 3i} = \dfrac{z - 1}{1 + i}$

d $\dfrac{3z - 2i}{2 + i} = \dfrac{2z + 5}{10 + 15i}$

More practice W.S. key

#1. a, 0 b. $2-2i$

c. $8+i$ d. $\frac{7}{29}+\frac{26}{29}i$

e 1 f 1

$i^5 \quad i^6 \quad i^7 \quad i^8 \qquad i^{2008} \quad i^{2009} \quad i^{2010} \quad i^{2011}$

e: $$\frac{(\overbrace{i+i^2+i^3+i^4}^{0})+(\overbrace{i+i^2+i^3+i^4}^{0})\cdots\cdots i^4(i+i^2+i^3)}{\underbrace{(\underbrace{i\cdot i^2\cdot i^3\cdot i^4}_{-1})(\underbrace{i\cdot i^2\cdot i^3\cdot i^4}_{-1})\cdots}_{(-1)^{502}=1}(i\cdot i^2\cdot i^3)}$$

$$=\frac{\cancel{i}+i^2+\cancel{i^3}}{i^1\cdot i^2\cdot i^3}=\frac{-1}{i^6}=\frac{-1}{i^4\cdot i^2}=\frac{-1}{-1}=\boxed{1}$$

#1. a. $\frac{-1}{5}+\frac{9}{5}i$ b, $-\frac{15}{17}-\frac{8}{17}i$ c. $\frac{-25}{39}+\frac{5}{39}i$

d. $\frac{19}{2}+\frac{15}{2}i$ e. $\frac{-13}{25}+\frac{84}{25}i$

#2. a $a=4,\ b=-3$ b. $a=4\ b=19$ c. $a=6,\ b=-5$

d. $a=0,\ b=4$

#5. a. $\frac{7}{10}-\frac{19}{10}i$ b. $2-i$ c. $\frac{31}{13}-\frac{12}{13}i$

d. $\frac{111}{505}+\frac{302}{505}i$