Goal: Using Logarithmic properties, Do problem solving. Key

IB Pre HL Growth and Decay **Now with Logs!!**

Warm up:

1) Solve for x: $\log_3 x + \log_3(x-2) = 1$

$x(x-2) = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$ → $x = 3$, $x \neq -1$ (x must be greater than 2)

2) Find a and b given that $2^a 8^b = \frac{1}{2}$ and $\frac{3^{-a}}{3^{b+1}} = 9$

$2^a \cdot 2^{3b} = 2^{-1}$ and $3^{-a} \cdot 3^{-(b+1)} = 3^2$ $\Rightarrow$ $a + 3b = -1$, $-a - b = 3$

$a + 3b = -1$ $\quad -a - b - 1 = 2$, $-a - b = 3$

$2b = 2 \Rightarrow a + 3 = -1$

$b = 1$ $\quad a = -4$

Example 1)

The weight of a radioactive substance remaining after t days is given by $W(t) = 2200(4)^{-.005t}$ grams.

a. What is the weight remaining after 6 days?

$W(6) = 2200(4)^{-.005(6)} \approx 2110.38 \approx 2110$ grams.

b. After how long with the remaining weight be 100 grams?

$100 = 2200(4)^{-0.005t}$ $\quad t = \frac{\log 22}{.005 \log 4} \approx$ 446 days

Example 2) $A = p\left(1 + \frac{i}{n}\right)^{nt}$ p: \$ invested i: Annual interest rate

a) Stan has \$5000 invest in an account that pays 5.2% annual interest rate compounded annually ($n=1$). Find how long (nearest month) it will take for her investment to reach \$20,000.

n: #s of compound

t: #s of yrs.

$\frac{20{,}000}{5} = 5000(1 + 0.052)^t$

$\log 4 = \log(1.052)^t \Rightarrow t = \frac{\log 4}{\log(1.052)} \approx 27.347 \text{yrs} \approx 329$ months

b) Stan has \$5000 invest in an account that pays 5.2% annual interest rate compounded monthly. Find how long (nearest month) it will take for her investment to reach \$20,000.

$n = 12$.

$\ln \frac{20}{5} = \ln\left(1 + \frac{0.052}{12}\right)^{12t}$

$\frac{\ln 4}{\ln(1.00433)} = 12t \Rightarrow t = 26.924$ yrs $\approx$ 324 months.