

Graphing Rational Functions with Oblique Asymptotes

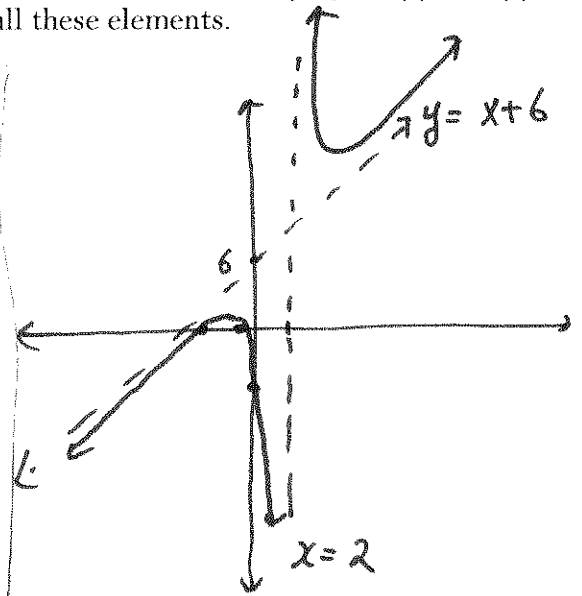
Perform a long or synthetic division and state the oblique asymptote(s), hole(s), x-intercept(s), and y-intercept. And then sketch the graph showing all these elements.

a. $f(x) = \frac{x^2 + 4x + 3}{x - 2}$

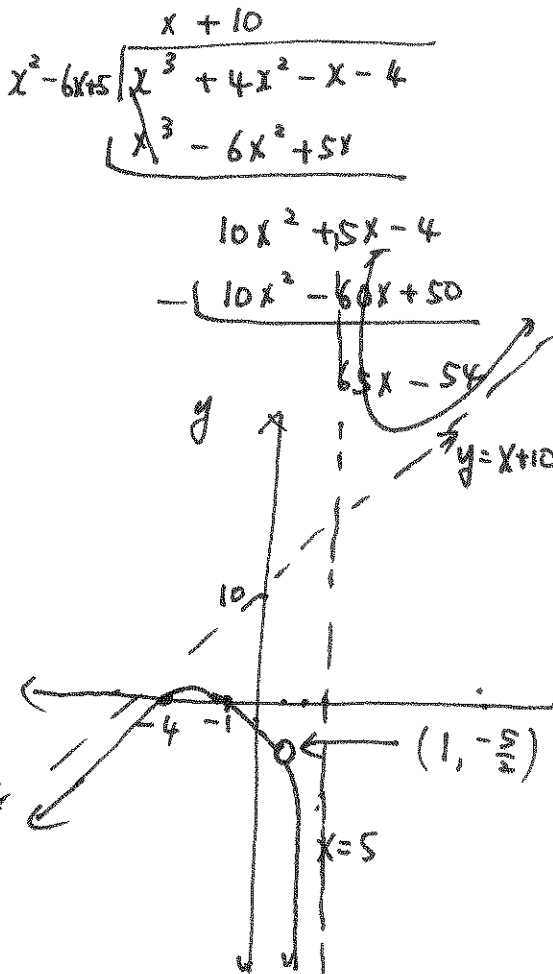
$$\begin{array}{r} 1 \quad 4 \quad 3 \\ 1 \quad 6 \quad 15 \\ \hline \end{array}$$

$f(x) = (x+6) + \frac{15}{x-2}$

Oblique Asymp: $y = x + 6$
 V.A: $x = 2$
 holes: None
 X-int: $(x^2 + 4x + 3) = 0$
 $(x+3)(x+1) = 0$
 $x = -1 \quad x = -3$
 y-int: $(-1, 0) \quad (-3, 0)$
 $y = \frac{3}{-2} \quad (0, -\frac{3}{2})$



b. $y = \frac{x^3 + 4x^2 - x - 4}{x^2 - 6x + 5} = \frac{x^2(x+4) - (x+4)}{(x-5)(x-1)} = \frac{(x+4)(x+1)(x-1)}{(x-5)(x-1)}$



Oblique Asymp: $y = x + 10$
 V.A: $x = 5$
 holes: $x = 1 \quad y = \frac{5 \cdot 2}{-4} = \frac{10}{-4} = -\frac{5}{2}$
 $(1, -\frac{5}{2})$
 X-int: $x = 4 \quad x = -1$
 $(4, 0) \quad (-1, 0)$
 y-int: $(0, -\frac{4}{5})$

key.

Practice WS:

key attached.

1. Without a calculator, sketch a complete graph of each function including the vertical and horizontal or oblique asymptotes, holes, x -intercepts, y -intercept and end behavior. Check your answers on your graphing calculator.

a. $f(x) = \frac{2x^2 - x - 6}{x + 4}$

b. $f(x) = \frac{3x^2 + 2x - 5}{x^2 + x - 2}$

c. $f(x) = \frac{x^3 + x^2 - 6x}{x^2 - x - 12}$

2. Find a rational function with the given information.

a. Vertical asymptotes; $x=3$ and $x=-1$.

Horizontal asymptote: $y = \frac{1}{2}$

Holes: none

x -intercepts: $(5, 0)$ and $(-2, 0)$

y -intercept: $(0, 5/3)$

$$\Rightarrow \frac{(x-5)(x+2)}{2(x-3)(x+1)} = f(x)$$

b. Vertical asymptotes; $x=1$ and $x=-1$.

Horizontal asymptote: $y=0$

Holes: none

x -intercepts: $(-3/2, 0)$

y -intercept: $(0, -1)$

$$\Rightarrow \frac{(2x+3)}{3(x-1)(x+1)} = f(x)$$

c. Vertical asymptotes; $x=1/2$.

Horizontal asymptote: $y=-1/2$

Holes: -1

x -intercepts: $(4, 0)$

y -intercept: $(0, -4)$

$$\Rightarrow \frac{-(x-4)(x+1)}{(2x-1)(x+1)} = f(x)$$