IB Pre HL Sum and Product of Quadratic Roots **Name:** Key **Period:** ____

No Calculators!

1. Without finding the roots, determine the sum and product of the roots of $2x^2+5x=20$.

Sum → $\alpha+\beta=\frac{-5}{2}$

product → $\alpha\beta=\frac{-20}{2}=-10$

2. The given equation $mx^2+(m-3)x+1-m=0$ is such that the sum of its roots is twice their product. Find the value of m and the two roots.

Sum → $\frac{-(m-3)}{m}$

product → $\frac{1-m}{m}$

$\Rightarrow \frac{-(m-3)}{m}=\frac{2(1-m)}{m}$ (Assume $m\neq 0$)

$-m+3=2-2m$

$\boxed{m=-1}$

$\Rightarrow -x^2-4x+20 \Rightarrow x^2+4x-2=0$

$\boxed{x=-2\pm\sqrt{6}}$

3. The given equation, $3x^2-5x+2=0$ has solutions a and b.

$a+b=\frac{5}{3}$

$a\cdot b=\frac{2}{3}$

a) Find the simplest quadratic equation with roots $\frac{1}{a}$ and $\frac{1}{b}$.

New Roots: $\frac{1}{a}, \frac{1}{b}$

Sum: $\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}=\frac{\frac{5}{3}}{\frac{2}{3}}=\frac{5}{2}$

Product: $\frac{1}{a}\cdot\frac{1}{b}=\frac{1}{ab}=\frac{3}{2}$

$\Rightarrow x^2-\frac{5}{2}x+\frac{3}{2}=0$

$\boxed{2x^2-5x+3=0}$

b) Find the simplest quadratic equation with roots $a+\frac{1}{b}$ and $b+\frac{1}{a}$.

New Roots: $a+\frac{1}{b}, b+\frac{1}{a}$

Sum: $a+\frac{1}{b}+b+\frac{1}{a}=a+b+\frac{1}{a}+\frac{1}{b}=\frac{5}{3}+\frac{5}{2}=\frac{10}{6}+\frac{15}{6}=\frac{25}{6}$

$\Rightarrow x^2-\frac{25}{6}x+\frac{25}{6}=0$

$\boxed{6x^2-25x+25=0}$

Product: $\left(a+\frac{1}{b}\right)\left(b+\frac{1}{a}\right)=ab+2+\frac{1}{ab}=\frac{2}{3}+2+\frac{3}{2}=\frac{4}{6}+\frac{9}{6}+2=\frac{13}{6}+\frac{12}{6}=\frac{25}{6}$.

b) Find the simplest quadratic equation with roots a^2 and b^2.

New Roots: a^2, b^2

Sum: a^2+b^2: $[a+b]^2=a^2+b^2+2ab=\left[\frac{5}{3}\right]^2$

$a^2+b^2=\frac{25}{9}-(2)\left(\frac{2}{3}\right)=\frac{25}{9}-\frac{12}{9}=\frac{13}{9}$

Product: $a^2\cdot b^2=\frac{4}{9}$

$\Rightarrow x^2-\frac{13}{9}x+\frac{4}{9}=0$

$\boxed{9x^2-13x+4=0}$

4. Write the quadratic equation, in standard form, with roots: $\frac{1}{3}$ *and* $-\frac{2}{5}$ and passing through the point (0,3).

$a[x^2 - (\text{sum})x + (\text{product})] = 0$

Sum: $\frac{1}{3} + (-\frac{2}{5}) = \frac{-1}{15}$, product: $(\frac{1}{3})(\frac{-2}{5}) = \frac{-2}{15}$.

$a[x^2 - (\frac{-1}{15})x - \frac{2}{15}] = 0 \Leftarrow (0,3)$

$3 = a[-\frac{2}{15}] \Rightarrow a = \frac{-45}{2}$.

$\Rightarrow -\frac{45}{2}(x^2 - (\frac{-1}{15})x + (\frac{-2}{15})) = 0 \Rightarrow \boxed{\frac{-45}{2}x^2 + \frac{3}{2}x + 3 = 0} \Rightarrow \boxed{-45x^2 + 3x + 6 = 0}$

6. The equation $2kx^2 + 2(k+4)x - 1 = 0$ has roots which are real. Find the possible values of k.

$\Delta = b^2 - 4ac \geq 0$

$a = 2k$, $b = 2(k+4)$, $c = -1$

$\Rightarrow [2(k+4)]^2 - (4)(2k)(-1) \geq 0$

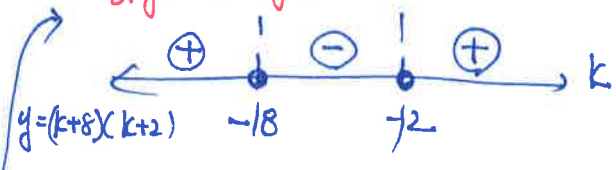
$= 4(k^2 + 8k + 16) + 8k \geq 0$

$4(k^2 + 8k + 16 + 2k) \geq 0$

$4(k^2 + 10k + 16) \geq 0$

$4(k+8)(k+2) \geq 0$

Sign diagram



$\Rightarrow \boxed{k: (-\infty, -8] \cup [-2, \infty)}$.

7. Given $y = -2x^2 + 5x - 2$;

a) Write the quadratic function in factored form and state the x-intercepts.

$y = -(x-2)(2x-1)$

x-int: $(2,0)$ and $(\frac{1}{2}, 0)$

b) Write the quadratic function in vertex form and state the coordinates of the vertex and its axis of symmetry.

$y = -2[x^2 - \frac{5}{2}x + \frac{25}{16}] - 2 + \frac{25}{8}$

$y = -2[x - \frac{5}{4}]^2 + \frac{9}{8}$

Vertex: $(\frac{5}{4}, \frac{9}{8})$ Axis of sym: $x = \frac{5}{4}$

c) Graph the function showing x-intercepts, y-intercept, and vertex.