Key.

Pre HL: The Sum and Product of the roots of Quadratic Equation. **Notes: September 8**

Warm UP

Find the discriminant and hence find the values of w for which the equation, $2x^2 - 5x + w = 0$ has i) a repeated root (one solution), ii) 2 distinct real roots, and iii) no real roots.

(i) $(-5)^2 - 4w \cdot 2 = 0 \Rightarrow 8w = 25 \Rightarrow \boxed{w = \frac{25}{8}}$

(ii) $(-5)^2 - 8w > 0 \Rightarrow \boxed{w < \frac{25}{8}}$

(iii) $\boxed{w > \frac{25}{8}}$

You know "If $ax^2 + bx + c = 0$ where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$."

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **means** $x_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ **or** $x_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$
Find $x_1 + x_2$ (sum of the roots) $\dfrac{-b + \cancel{\sqrt{b^2 - 4ac}}}{2a} + \dfrac{-b - \cancel{\sqrt{b^2 - 4ac}}}{2a} = \dfrac{-\cancel{2}b}{\cancel{2}a} = \boxed{\dfrac{-b}{a}}$
Find $x_1 \bullet x_2$ (product of the roots) $(a+b)(a-b) = a^2 - b^2$ $\left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$ $= \dfrac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2} = \dfrac{\cancel{b^2} - (\cancel{b^2} - 4ac)}{4a^2}$ $= \dfrac{\cancel{4}ac}{\cancel{4}a^2} = \boxed{\dfrac{c}{a}}$

$\therefore$ *if* $ax^2 + bx + c = 0$ **where** $a \neq 0$**, then the sum of the roots is** $-\dfrac{b}{a}$ **and the product of the roots is** $\dfrac{c}{a}$**.**

a

$x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0$

Let's apply what you have learned.

Ex1) Find the sum and product of the roots of $25x^2 - 20x + 1 = 0$. Validate if your answer by solving the quadratic.

$\overline{25x^2 - 20x + 1 = 0}$
25

$$x^2 - \frac{20}{25}x + \frac{1}{25} = 0$$

sum : $\frac{20}{25} = \frac{4}{5}$

product: $\frac{1}{25}$.

Ex2) The quadratic equation $x^2 + (k-3)x + (2-k) = 0$ has one root which is three less than other root. Find all possible values of k and the two roots.

$\boxed{x_1}$ $\qquad x_2 = \boxed{x_1 - 3}$

① $(x_1)(x_1 - 3) = 2 - k$

② $x_1 + (x_1 - 3) = -(k-3) \Rightarrow 2x_1 - 3 = 3 - k$

$2x_1 - 6 = -k$

$k = 6 - 2x_1$

(continue on the back!

Practice1) $3x^2 + x - 1 = 0$ has roots p and q. Find $p^2 + q^2$.

$p + q = \frac{-1}{3}$

$p \cdot q = -\frac{1}{3}$.

$(p+q)^2 = \left(\frac{-1}{3}\right)^2$

$p^2 + q^2 + 2pq = \frac{1}{9}$

$p^2 + q^2 = \frac{1}{9} - 2 \cdot pq = \frac{1}{9} + \frac{2 \cdot 3}{3 \cdot 3} = \boxed{\frac{7}{9}}$

Practice2) The quadratic equation $x^2 + (k-12)x + (k+7) = 0$ has the roots of consecutive positive integers. Find possible values of k and the two roots.

Roots : 3, 4 $\qquad \boxed{k = 5}$

$x_1 = a \qquad x_2 = a + 1$

$a(a+1) = k + 7$

$a + (a+1) = 12 - k$.

$2a + 1 = 12 - k \Rightarrow -k = 2a - 11 \qquad k = 11 - 2a$.

$a^2 + a = 11 - 2a + 7$

$a^2 + 3a - 18 = 0 = (a+6)(a-3) =$

$+6$
-3

$a = 3 \qquad a \cancel{= -6}$

Ex2)

$x_1(x_1-3) = 2-(6-2x_1)$

$(x_1)^2 - 3x_1 = 2-6+2x_1$

$(x_1)^2 - 5x_1 + 4 = 0$

$\boxed{x_1 = 4}$ $\quad x_1 = 1 \quad \Rightarrow k = 6-2(4)$

$\boxed{x_2 = 1} \quad = \boxed{-2}$

OR. $x_1 = 1 \quad \Rightarrow k = 6-2(1)$

$x_2 = 1-3 = -2$.

$= 4$

Two possible Answers for value of k.

$k=-2$ Roots: 1 & 4 $[x^2-5x+4=0]$

$k=4$ Roots: 1 & -2 $[x^2+x-2=0]$