IB Pre HL Rational Roots (or Zeros) Theorem Notes Name: ______________________

Rational Zeros Theorem:

If f is a polynomial function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, with degree $n \geq 1$, integer coefficients, and $a_0 \neq 0$, then every rational zero of f has the form $\frac{p}{q}$, where

- p and q have no common factors other than ± 1,
- p is an integer factor of the constant term a_0, and
- q is an integer factor of the leading coefficient a_n.

Corollary If the leading coefficient a_n is 1, then any rational zeros of f are integer factors of the constant term a_0.

Example:

List all possible rational zeros of $h(x) = x^3 - 5x^2 - 17x - 6$. Then determine which, if any, are zeros.

The leading coefficient is 1 and the constant term is -6.

Possible rational zeros: $\dfrac{\text{Factors of } -6}{\text{Factors of } 1} = \dfrac{\pm1, \pm2, \pm3, \pm6}{\pm1}$ or $\pm1, \pm2, \pm3, \pm6$

By synthetic substitution, you can determine that $x = -2$ is a rational zero.

-2	1	-5	-17	-6
		-2	14	6
	1	-7	-3	0

$\Rightarrow (x+2)(x^2-7x-3)$

The depressed polynomial is $x^2 - 7x - 3$. You can use the quadratic formula to find the two irrational roots.

Let's do another example together:

Given the polynomial $f(x) = 6x^4 + 17x^3 + 10x^2 - 7x - 6$, find all roots.

$p: \pm1, \pm2, \pm3, \pm6$

$q: \pm1, \pm2, \pm3, \pm6$

$\Rightarrow \frac{p}{q} \in \{\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}, \pm\frac{3}{2}\}$

1	6	17	10	-7	-6
		6	23	33	26
	6	23	33	26	20

-1	6	17	10	-7	-6
		-6	-11	1	6
-1	6	11	-1	-6	0
		-6	-5	6	
	6	5	-6	0	

$\Rightarrow (x+1)(x+1)(6x^2+5x-6)$

$2x \quad +3$

$3x \quad -2$

$0 = (x+1)(x+1)(2x+3)(3x-2)$

$x = -1, \quad x = -\frac{3}{2} \quad x = \frac{2}{3}$