

Example 1) $9 - 12 \sin \theta = 4 \cos^2 \theta$ $0 < \theta \leq 2\pi$

$\cos^2 \theta = 1 - \sin^2 \theta$

$9 - 12 \sin \theta = 4(1 - \sin^2 \theta)$

$9 - 12 \sin \theta = 4 - 4 \sin^2 \theta$

$-4 + 4 \sin^2 \theta \quad -4 + 4 \sin^2 \theta$

$4 \sin^2 \theta - 12 \sin \theta + 5 = 0$

$2 \sin \theta \quad -1$

$2 \sin \theta \quad -5$

$\Rightarrow (2 \sin \theta - 1)(2 \sin \theta - 5) = 0$



$\sin \theta = \frac{1}{2}$

~~$\sin \theta = \frac{5}{2}$~~

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Example 2) $\cos 2\theta - \sin \theta = 1$ $0 < \theta \leq 2\pi$

$\cos 2\theta = 1 - 2 \sin^2 \theta$

$-2 \sin^2 \theta - \sin \theta = 0$

$-2 \sin^2 \theta - \sin \theta = 0$

$\Rightarrow 2 \sin \theta + \sin \theta = 0$

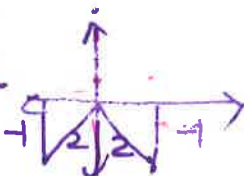
$\sin \theta (2 \sin \theta + 1) = 0$

$\sin \theta = 0$

$\sin \theta = -\frac{1}{2}$ $0 < \theta \leq 2\pi$

$\theta = 2\pi, \pi$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



Example 4) Find the exact value of $\tan A$ if $\tan 2A = \frac{21}{20}$ and A is obtuse.

$\tan 2A = \frac{21}{20}$

$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$90^\circ < \text{obtuse} < 180$

~~$\frac{2 \tan A}{1 - \tan^2 A} = \frac{21}{20}$~~

$\Rightarrow 40 \tan A = 21 - 21 \tan^2 A$

$+ 21 \tan^2 A - 21 \quad -21 \quad + 21 \tan^2 A$
 $= 0$

$21 \tan^2 A + 40 \tan A - 21 = 0$

$3 \quad + 7$

$7 \quad - 3$

$\Rightarrow (3 \tan A + 7)(7 \tan A - 3) = 0$

$\tan A = -\frac{7}{3}$

~~$\tan A = \frac{3}{7}$~~

$A = \tan^{-1}(-\frac{7}{3}) \approx 113^\circ$

Worksheet one continues (do your work in separate paper).

Solve for $0 < \theta \leq 2\pi$, giving your answers as exact values:

7. $\cot \theta \cos \theta = \cos \theta$

8. $\cos \theta \csc \theta = 2 \cos \theta$

9. $\sin^2 \theta + 3 \cos^2 \theta = 0$

10. $\sec^2 \theta - 2 \tan^2 \theta = 0$

11. $\cos^2 \theta - \sin^2 \theta + \sin \theta = 0$

