Final Questions

① $f(x) = 2x^3 + ax^2 - 22x - 24$

If $(2x+3)$ is a factor, find a where $a \in R$ and the other factors

$2\left(-\frac{3}{2}\right)^3 + a\left(-\frac{3}{2}\right)^2 - 22\left(-\frac{3}{2}\right) - 24 = 0$

$-\frac{24}{4} + \frac{9}{4}a + 33 - 24 = 0$

$-\frac{27}{4} + 9 + \frac{9}{4}a = 0$

$\frac{9}{4}a = \frac{-9}{4}a$

$a = -1$

$f(x) = 2x^3 - x^2 - 22x - 24$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & -22 & -24 \\ & \downarrow & -3 & 6 & 24 \\ \hline & 2 & -4 & -16 & 0 \end{array}$$

$(2x^2 - 4x - 16) \div 2$

$x^2 - 2x - 8$

$\boxed{(x-4)(x+2)(2x+3)}$

② When $x^3 + 2x^2 + ax + b$ is divided by $x-1$ the remainder is 4, and when divided by $x+2$ the remainder is 16. Find constants $\underline{a}$ and $\underline{b}$

$x^3 + 2x^2 + ax + b$

$p(1) = 1 + 2 + a + b$

$a + b = 1$

$p(-2) = (-2)^3 + 2(-2)^2 + a(-2) + b$

$16 = -8 + 8 - 2a + b$

$16 = -2a + b$

$$\begin{array}{rl} & -2a + b = 16 \\ - & a + b = 1 \\ \hline & -3a = 15 \end{array} \Rightarrow \boxed{a = -5}$$

$-5 + b = 1$

$\boxed{b = 6}$

③ Perform the division $\frac{x^4 + 2x^2 - 1}{x+3}$ through the synthetic division method.
→ Write it in the form: $P(x) = Q(x)D(x) + R(x)$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 2 & 0 & -1 \\ & \downarrow & -3 & 9 & -33 & 99 \\ \hline & 1 & -3 & 11 & -33 & 98 \end{array}$$

$\boxed{(x^3 - 3x^2 + 11x - 33)(x+3) + 98}$

Andreea D, Joseph L, Jeslyn C, Puneet P

① Divide synthetically and find the remainder - Andreea

$(x^3+x^2+56) \div (x+1)$

$$\begin{array}{r|rrr} & 1 & 1 & 56 \\ -1 & & -1 & 0 \\ \hline & 1 & 0 & 56 \end{array}$$

$(x^2)+\left(56/_{x-1}\right)$

② The polynomial $P(x)=x^3+rx^2+5x-9$ has one root that is equal to $(-2-\sqrt{5}i)$, find the other roots. - Puneet

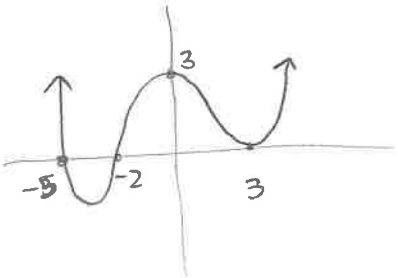
$(-2-\sqrt{5}i)(-2+\sqrt{5}i) \Rightarrow x^2+4x+9$

$(-2-\sqrt{5}i)^3+(-2-\sqrt{5}i)^2 r+5(-2-\sqrt{5}i)-9=0$

$r=3$

$(x-1)(x+2+\sqrt{5}i)(x+2-\sqrt{5}i)(x-1)$

③ Find the equation from the graph



$3=a(5)(2)(9)$

$3=a(90)$

$a=\frac{1}{30}$

$P(x)=\frac{1}{30}(x+5)(x+2)(x-3)^2$

Write in the form $P(x) = Q(x)\,D(x) + R$ Ronit Dasgupta

1. $\dfrac{x^2 - 3x + 6}{x-4}$

$$\begin{array}{r|rrr} 4 & 1 & -3 & 6 \\ & \downarrow & 4 & 4 \\ \hline & 1x & 1 & 10 \end{array}$$

$\boxed{x + 1 + \frac{10}{x-4}} \rightarrow \boxed{(x+1)(x-4) + 10}$

$\boxed{P(x) = (x+1)(x-4) + 10}$

Find the sum and product of the roots of :

2. $2x^2 - 3x + 4 = 0$

sum: $\frac{-b}{a} \rightarrow \frac{-(-3)}{2} = \boxed{\frac{3}{2}}$

product: $\frac{c}{a} \rightarrow \frac{4}{2} = \boxed{2}$

3. Find the constant k and hence factorise the polynomial if :

$2x^3 + x^2 + kx - 4$ has the factor $(x+2)$

$2(-2)^3 + (-2)^2 + k(-2) - 4 = 0$

$16 + 4 - 2k - 4 = 0$

$-2k - 16 = 0$

$\frac{-2k}{-2} = \frac{16}{-2}$

$\boxed{k = -8}$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -8 & -4 \\ & \downarrow & -4 & 6 & 4 \\ \hline & 2x^2 & -3x & -2 & 0 \end{array}$$

$\nwarrow$ remainder

$2x^2 - 3x - 2 + \frac{0}{x+2}$

$\boxed{P(x) = (x+2)(2x+1)(x-2)}$

OUR 3 QUESTIONS

Anjali
Enrico
Simon

1) $P(x) = 3x^2 + 9x + 10$
find the quotient and remainder when p(x) is divided by (x+4)

$$\begin{array}{r|l} & 3x - 3 \\ \hline x+4 & 3x^2 + 9x + 10 \\ & 3x^2 + 12x \\ \hline & -3x + 10 \\ & -3x - 12 \\ \hline & 22 \end{array}$$

$$\boxed{= 3x - 3 + \frac{22}{x+4}}$$

2) Find all possible cubic polynomials with zeroes

$\pm 6,\ x+i$ - has to have x - i for the equation to work, therefore...

$(x+6)(x-6)(x-(x+i))$

$(x-6)(x+6)(x-x-i)$

$(x^2-36)(-i)$

$= -x^2 i + 36i$

$(x+6)(x-6)(x-(x+i))(x-(x-i))$

$(x^2-36)(x^2-2x+2)$

$(x^4-2x^3+2x^2-36x^2+72x-72)$

$(x^4-2x^3 \cancel{+} -34x^2+72x-72)$

3) A polynomial is $P(x) = -2x^4 + 7x^3 - cx^2 + dx + e$. y intercept is 45 the graph passes the x-axis at (4,0) and (-1,0) Find (+d) there are also only 2 roots? Clarity

Roots: $4 \pm i$?

$P(4) = 0 \quad -2(4)^4 + 7(4)^3 - c(4)^2 + d(4) + 45$ Show Work

[redacted]

$$\frac{-16c + 4d}{4} = \frac{-19}{4} \rightarrow -4c + d = -4.75$$

Chapter 6 Review Question Assessment

*highlighted is our questions, we ran out of time, the sub said it was ok

YeonJun Ko
Period 3

1) When $f(x)$ is divided by x^2-x+6, the quotient is x^2+2x+1 with remainder $ax+b$. When $f(x)$ is Divided by $(x+2)$, the remainder is -8 and when Divided $(x-3)$, the remainder is 2. Find $f(x)$.

Solution) $f(x)=(x^2-x+6)(x^2+2x+1)+ax+b$

$f(-2)=(4+2+6)(4-4+1)-2a+b,\ 12-2a+b,\ -2a+b=-12$

$f(3)=(9-3+6)(9+6+1)+3a+b,\ (12)(16)+3a+b,\ 3a+b=-192$

Don't forget that the reminder is -8 and 2 here

$$\begin{array}{l} -2a+b=-12 \\ -\ \ 3a+b=-192 \\ \hline -5a\ \ \ =180 \end{array}$$

$a=-36$

$b=-84$

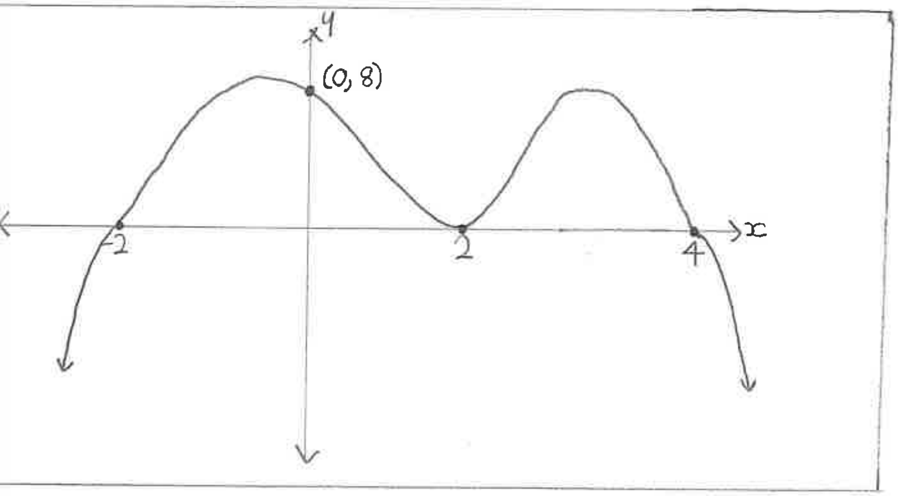
$f(x)=(x^4+\underline{2x^3}+\underline{x^2}-\underline{x^3}-\underline{2x^2}-x+\underline{6x^2}+12x+6)-36x-84$

$f(x)=x^4+x^3+5x^2+11x+6-36x-84$

$f(x)=x^4+x^3+5x^2-25x-78$

$$\boxed{\text{Answer} = f(x)=x^4+x^3+5x^2-25x-78}$$

2) Find the $f(x)$ equation of the graph.



Solution) $n=\text{even}$, ↓ ↓

Roots = $x=-2$

$x=4$

$x=2$ (repeated)

⇓

$(x+2)(x-4)(x-2)^2=f(x)$

$f(x)=(x^2-2x-8)(x^2-4x+4)a$

$f(x)=(x^4\underline{-4x^3}+\underline{4x^2}\underline{-2x^3}+\cancel{\underline{8x^2}}-8x-\cancel{\underline{8x^2}}+32x-32)a$

$f(x)=(x^4-6x^3+4x^2+24x-32)a$

$8=0-0+0+0-32a$

$8=-32a$

$-\frac{1}{4}=a$

$$\boxed{\text{Answer} = f(x)=-\tfrac{1}{4}(x^4-6x^3+4x^2+24x-32)}$$

Circled is chosen Questions
-approved by sub teacher

3) $f(x) = 4x^3 + ax^2 + 2x + b$. $f(1) = 10$, $f(2) = 51$. If the roots of the equation are r, s, t, find $r + s + t + rst$.

$f(1) = 10$: $f(1) = 4 + a + 2 + b = 10$

$a + b = 4$ ①

$f(2) = 51$: $f(2) = 32 + 4a + 4 + b = 49$

$4a + b = 13$ ②

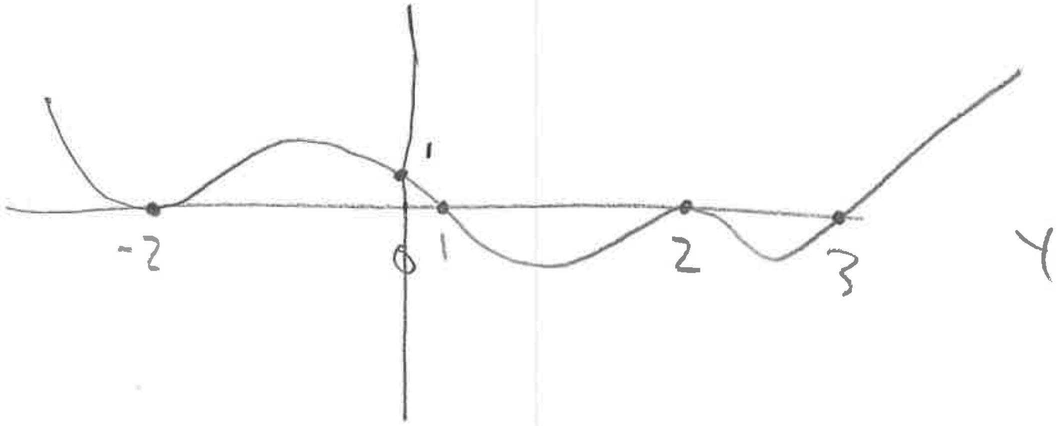
② - ① $3a = 9 \rightarrow \underline{a = 3} \rightarrow \underline{b = 1}$

$-\frac{a}{4} = r + s + t \Rightarrow r + s + t = -\frac{3}{4}$

$-\frac{b}{4} = rst \Rightarrow rst = -\frac{1}{4}$

$r + s + t + rst = -\frac{3}{4} - \frac{1}{4} = \boxed{-1}$

1. Find equation for



$$Y = \frac{1}{48}(x+2)^2(x-1)(x-2)^2(x-3)$$

$$1 = 4(-1)(4)(-3)$$

$$\frac{1}{48} = \frac{48a}{48} \qquad \frac{1}{48} = a$$

2. $x^3 + 2x^2 + 4x + 10 \div (x+2)$

$$\begin{array}{r|rrrr} & 1 & 2 & 4 & 10 \\ -2 & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 2 \end{array}$$

$$\boxed{x^2 + 4 + \frac{2}{x+2}}$$

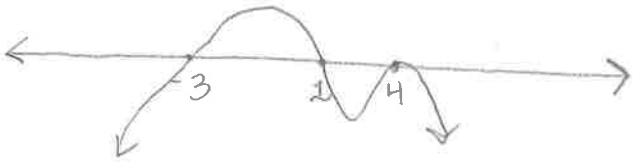
1. Sketch the graph of the polynomial showing x-intercepts and end behavior

$-3(x-4)^2(x+3)(x-1)$

Answer:

$n=4$

$C0<0$



2. Divide: $\frac{x^3-6x^2+3x-4}{x-1}$

A:

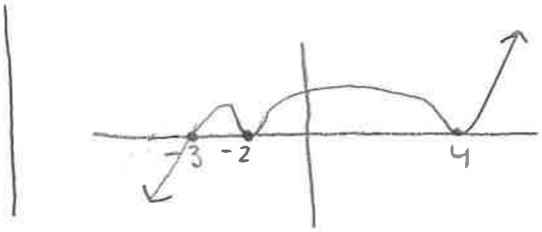
$$\begin{array}{r|rrrr} 1 & 1 & -6 & 3 & -4 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & \big| -6 \end{array}$$

$$\boxed{x^2-5x-2-\frac{6}{x-1}}$$

1. Sketch the graph of the polynomial showing x-intercepts and end behavior.

$(x+2)^2\ (x+3)\ (x-4)^2$

solution:



2. A real polynomial has the form $4x^4-16x^3+cx^2+dx+e$. The graph has a y-intercept of 240. It cuts the x axis at 2 and 6 and does not touch anywhere else. Suppose the other 2 roots are m ± ni. Find m and n.

solution:

Sum: 4

product: 60

$$2+6+m+\cancel{n}i+m-\cancel{n}i=4$$
$$2+6+2m=4$$
$$8+2m=4$$
$$2m=-4$$
$$\boxed{m=-2}$$

$$(2)(6)(m+ni)(m-ni)=60$$
$$\frac{(12)(m^2+n^2)}{12}=\frac{60}{12}$$
$$m^2+n^2=5$$
$$(-2)^2+n^2=5$$
$$4+n^2=5$$
$$\sqrt{n^2}=\sqrt{1}$$
$$\boxed{n=\pm1}$$

1. Divide $\frac{x^3-6x^2+3x-4}{x-1}$

solution:

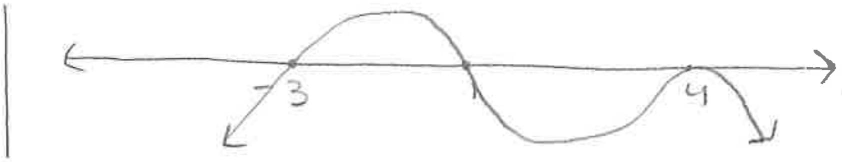
$$\begin{array}{c|cccc} 1 & 1 & -6 & 3 & -4 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & -6 \end{array}$$

$$\boxed{x^2-5x-2-\frac{6}{x-1}}$$

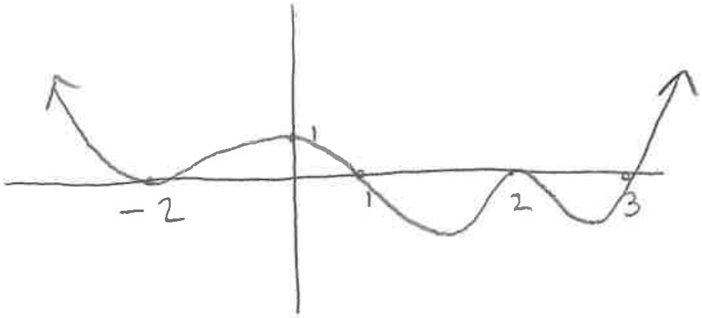
2. Sketch the graph of the polynomial showing x-intercepts and end behavior.

$-3(x-4)^2(x+3)(x-1)$

solution:



3. Find equation for



solution:

$y = a(x+2)^2(x-1)(x-2)^2(x-3)$

$1 = a(4)(-1)(4)(-3)$

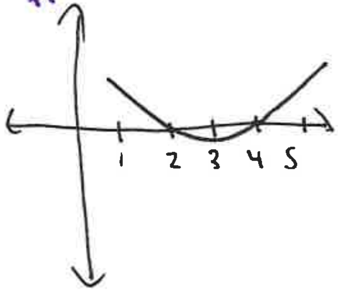
$\frac{1}{48} = \frac{a(48)}{48}$

$\frac{1}{48} = a$

$$\boxed{y = \frac{1}{48}(x+2)^2(x-1)(x-2)^2(x-3)}$$

Sarah Zhong, Dina Garber, Nazneen Poonawala, Nirmal Kumar

#3



Question: This is a graph of $x^5-3x^4-11x^3+27x^2+10x-24$, but only th 1st quadrant is visible. Fully Factorize.

Solution:

from th graph you get $(x-2)(x-4)$

①
$$\begin{array}{r|l} & x^3+3x^2-x-3 \\ \hline x^2-6x+8 & x^5-3x^4-11x^3+27x^2+10x-24 \\ & \underline{x^5-6x^4+8x^3} \\ & \quad 3x^4-19x^3+27x^2 \\ & \quad \underline{3x^4-18x^3+24x^2} \\ & \qquad -x^3+3x^2+10x \\ & \qquad \underline{-(-x^3+6x^2-8x)} \\ & \qquad\quad -3x^2+18x-24 \\ & \qquad\quad \underline{-3x^2+18x-24} \\ & \qquad\qquad 0 \end{array}$$

② x^3+3x^2-x-3

$$\frac{\text{factors of } -3}{\text{factors of } 1} \quad \frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$$

guess and checkuntil

③ $f(1)=1+3-1-3=0$

$x=1 \rightarrow (x-1)$

④
$$\begin{array}{r|rrrr} 1 & 1 & 3 & -1 & -3 \\ & & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

⑤ x^2+4x+3

$(x+3)(x+1)$

$(x-2)(x-4)(x-1)(x+3)(x+1)$

Find constants a and b given that $2x^3 + ax^2 + bx + 5$ has factors $(x-1)$ and $(x+5)$

#2

$2x^3 + ax^2 + bx + 5$ roots: $x = 1$
$x = -5$

① $2(1)^3 + a(1)^2 + b + 5 = 0$

$2 + a + b + 5 = 0$

① $\boxed{a + b = -7}$ $a = -b - 7$

② $2(-5)^3 + a(-5)^2 + b(-5) + 5 = 0$

$-250 + 25a - 5b + 5 = 0$

$25a - 5b = 245$

② $\boxed{5a - b = 49}$

$5(-b-7) - b = 49$

$-5b - 35 - b = 49$

$-6b = 84$

$\boxed{b = -14}$

$a - 14 = -7$

$\boxed{a = 7}$

Dina Garber, Nazneen Poonawala, Nirmal Marimuthu, Sarah Zhong

\#1

For the purely imaginary equation $ax^3+(a+1)x^2+10x+15, a \in R$. Find the value of a and all zeroes.

① imaginary zero = bi   zeroes = bi, -bi

② sum = 0. product $= -b^2i^2 = b^2$

$\pm bi$ comes from x^2+b^2

③ $$ax^3+(a+1)x^2+10x+15=(x^2+b^2)(ax+\frac{15}{b^2})$$
$$= ax^3+(\frac{15}{b^2})x^2+ab^2x+15$$

④ ① $a+1=\frac{15}{b^2}$

② $ab^2=10$

⑤ $ab^2+b^2=15$

$10+b^2=15$

$b^2=5$

$b=\pm\sqrt{5}$

⑥ $b^2=5$

$5a=10$

$a=2$

⑦ $ax+\frac{15}{b^2}=2x+3$

⑧ $\boxed{a=2 \text{ zeroes are } \pm i\sqrt{5}, \frac{-3}{2}}$

Nazneen Poonawala, Sarah Zhong, Dina Garber, Nirmal Marimuthu

Cathy Wu

1) Given the polynomial $P(x) = x^3 - 2x^2 - ax + b$ and the roots $(x+3)$ and $(x-1)$, find a and b as well as the other roots.

$$P(-3) = (-3)^3 - 2(-3)^2 - a(-3) + b = 0$$

$$-27 - 18 + 3a + b = 0$$

$$3a + b = 45$$

$$P(1) = (1)^3 - 2(1)^2 - a(1) + b = 0$$

$$1 - 2 - a + b = 0$$

$$-a + b = 1$$

$$\begin{array}{r} 3a+b = 45 \\ -\quad -a+b = 1 \\ \hline 4a = 44 \end{array}$$

$$\boxed{a = 11}$$

$$3(11) + b = 45$$

$$33 + b = 45$$

$$\boxed{b = 12}$$

$$P(x) = x^3 - 2x^2 - 11x + 12$$

$$(x+3)(x-1)$$

$$x^2 + 2x - 3$$

$$\begin{array}{r|l} & x - 4 \\ \hline x^2 + 2x - 3 & x^3 - 2x^2 - 11x + 12 \\ - & x^3 + 2x^2 - 3x \\ \hline & -4x^2 - 8x + 12 \\ & -4x^2 - 8x + 12 \\ \hline & 0 \end{array}$$

$$\boxed{P(x) = (x+3)(x-1)(x-4)}$$

2) Given the polynomial $P(x) = x^3 - 5x^2 + 28x - 40$ and one of the roots is $2+4i$, find the remaining roots.

$$(x - (2+4i))(x - (2-4i))$$

$$(x-2)^2 + 16$$

$$x^2 - 4x + 4 + 16$$

$$x^2 - 4x + 20$$

$$\begin{array}{r|l} & x - 2 \\ \hline x^2 - 4x + 20 & x^3 - 6x^2 + 28x - 40 \\ - & x^3 - 4x^2 + 20x \\ \hline & -2x^2 + 8x - 40 \\ & -2x^2 + 8x - 40 \\ \hline & 0 \end{array}$$

$$P(x) = (x - (2+4i))(x - (2-4i))(x-2)$$

1) Given the polynomial $2x^4+7x^3-6x^2-17x+14$, find all roots.

Selected

$$\frac{\pm1, \pm2, \pm7, \pm14}{\pm1, \pm2} \Rightarrow 1, -1, 2, -2, 7, -7, 14, -14, \frac{1}{2}, \frac{-1}{2}, \frac{7}{2}, \frac{-7}{2}$$

$\Downarrow$ testing

$$f(x)=2x^4+7x^3-6x^2-17x+14=0$$

$$\boxed{x=1, -2, \frac{-7}{2}}$$

4th degree, 4 roots are needed

---

2) Find a 4th degree polynomial with zeros $\pm\sqrt{7}$ and $5+i$.

$(x+\sqrt{7})(x-\sqrt{7})(x-(5+i))(x-(5-i))$

$(x^2-7)(x^2-(5+i)x-(5-i)x+26)$

$(x^2-7)(x^2-10x+26)$

$\boxed{x^4-20x^3-x^2+70x-42}$

Since Product $= a^2+b^2$

5^2+i^2

$25+1$

26

Bryce Wang pd 3