1. Divide $\frac{x^3-6x^2+3x-4}{x-1}$

solution:

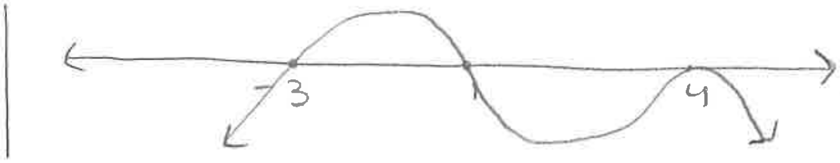
$$\begin{array}{r|rrrr} 1 & 1 & -6 & 3 & -4 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & -6 \end{array}$$

$$\boxed{x^2-5x-2-\frac{6}{x-1}}$$

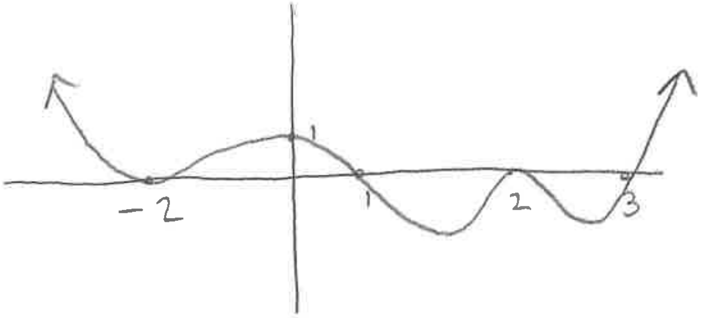
2. Sketch the graph of the polynomial showing x-intercepts and end behavior.

$-3(x-4)^2(x+3)(x-1)$

solution:



3. Find equation for



solution:

$y=a(x+2)^2(x-1)(x-2)^2(x-3)$

$1=a(4)(-1)(4)(-3)$

$\frac{1}{48}=\frac{a(48)}{48}$

$\frac{1}{48}=a$

$$\boxed{y=\frac{1}{48}(x+2)^2(x-1)(x-2)^2(x-3)}$$

For the purely imaginary equation

#1 $ax^3+(a+1)x^2+10x+15, a \in R$. Find the value of a and all zeroes.

① imaginary zero = bi   zeroes = bi, -bi

② sum = 0. product = $-b^2i^2 = b^2$

±bi comes from x^2+b^2

③ $ax^3+(a+1)x^2+10x+15 = (x^2+b^2)(ax+\frac{15}{b^2})$

$= ax^3+(\frac{15}{b^2})x^2+ab^2x+15$

④

① $a+1 = \frac{15}{b^2}$

② $ab^2=10$

⑤

$ab^2+b^2=15$

$10+b^2=15$

$b^2=5$

$b=\pm\sqrt{5}$

⑥

$b^2=5$

$5a=10$

$a=2$

⑦ $ax+\frac{15}{b^2}=2x+3$

⑧ $\boxed{a=2 \quad \text{zeroes are } \pm i\sqrt{5}, -\frac{3}{2}}$

Nazneen Poonawala, Sarah Zhang,
Dina Garber, Nirmal Marimuthu

Find constants a and b given that $2x^3+ax^2+bx+5$ has factors $(x-1)$ and $(x+5)$

#2

$2x^3+ax^2+bx+5$ roots: $x=1$
$x=-5$

① $2(1)^3+a(1)^2+b+5=0$

$2+a+b+5=0$

① $\boxed{a+b=-7}$ $a=-b-7$

② $2(-5)^3+a(-5)^2+b(-5)+5=0$

$-250+25a-5b+5=0$

$25a-5b=245$

② $\boxed{5a-b=49}$

$5(-b-7)-b=49$

$-5b-35-b=49$

$-6b=84$

$\boxed{b=-14}$

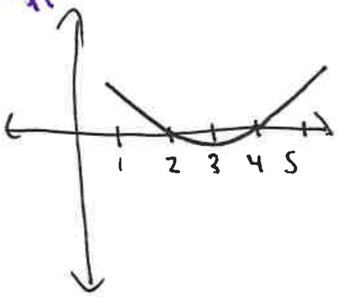
$a-14=-7$

$\boxed{a=7}$

Dina Garber, Nazneen Poonawala, Nirmal Marimuthu, Sarah Zhong

Sarah Zhong, Dina Garber, Nazneen Poorawala, Nirmal Kumar

#3



Question: This is a graph of $x^5-3x^4-11x^3+27x^2+10x-24$, but only the 1st quadrant is visible. Fully Factorize.

Solution:

from the graph you get $(x-2)(x-4)$

①
$$\begin{array}{r}
x^3+3x^2-x-3 \\
x^2-6x+8 \overline{)\, x^5-3x^4-11x^3+27x^2+10x-24} \\
\underline{x^5-6x^4+8x^3} \\
3x^4-19x^3+27x^2 \\
\underline{3x^4-18x^3+24x^2} \\
-x^3+3x^2+10x \\
\underline{-(-x^3+6x^2-8x)} \\
-3x^2+18x-24 \\
\underline{-3x^2+18x-24} \\
0
\end{array}$$

② x^3+3x^2-x-3

$$\frac{\text{factors of } -3}{\text{factors of } 1} \quad \frac{\pm1, \pm3}{\pm1} = \pm1, \pm3$$

guess and check until

③ $f(1)=1+3-1-3=0$

$x=1 \rightarrow (x-1)$

④
$$\begin{array}{r|rrrr}
1 & 1 & 3 & -1 & -3 \\
 & & 1 & 4 & 3 \\
\hline
 & 1 & 4 & 3 & 0
\end{array}$$

⑤ x^2+4x+3

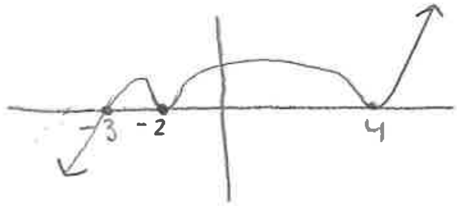
$(x+3)(x+1)$

$(x-2)(x-4)(x-1)(x+3)(x+1)$

1. Sketch the graph of the polynomial showing x-intercepts and end behavior.

$(x+2)^2 (x+3) (x-4)^2$

solution:



2. A real polynomial has the form $4x^4-16x^3+cx^2+dx+e$. The graph has a y-intercept of 240. It cuts the x axis at 2 and 6 and does not touch anywhere else. Suppose the other 2 roots are m ± ni. Find m and n.

solution:

sum: 4

product: 60

$$2+6+m+\cancel{ni}+m-\cancel{ni}=4$$

$$2+6+2m=4$$

$$8+2m=4$$

$$2m=-4$$

$$\boxed{m=-2}$$

$$(2)(6)(m+ni)(m-ni)=60$$

$$\frac{(12)(m^2+n^2)}{12}=\frac{60}{12}$$

$$m^2+n^2=5$$

$$(-2)^2+n^2=5$$

$$4+n^2=5$$

$$\sqrt{n^2}=\sqrt{1}$$

$$\boxed{n=1}$$

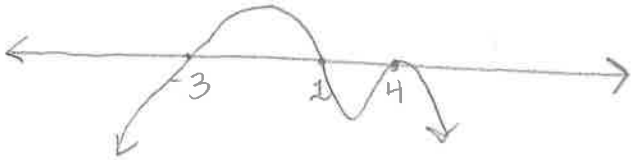
1. Sketch the graph of the polynomial showing x-intercepts and end behavior

$-3(x-4)^2(x+3)(x-1)$

Answer:

$n=4$

$CO<0$



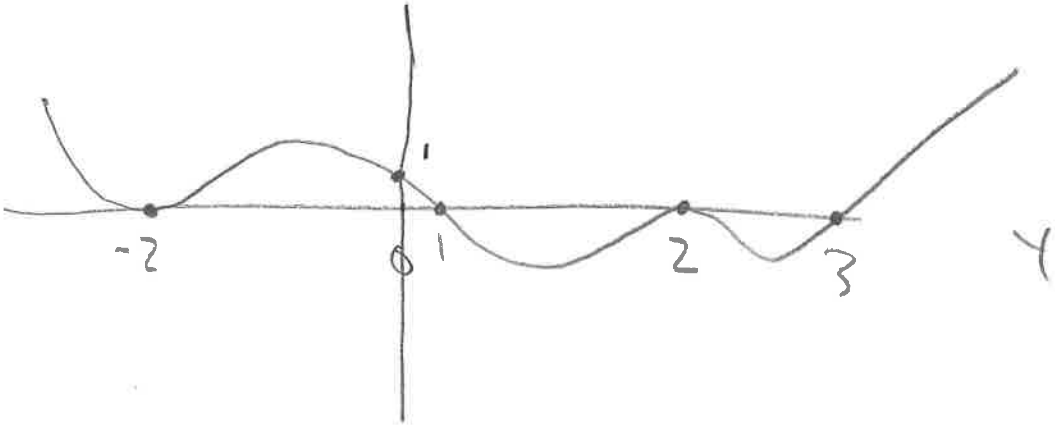
2. Divide: $\frac{x^3-6x^2+3x-4}{x-1}$

A:

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 3 & -4 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & -6 \end{array}$$

$$\boxed{x^2-5x-2-\frac{6}{x-1}}$$

1. Find equation for



$Y = \frac{1}{48}(x+2)^2(x-1)(x-2)^2(x-3)$

$1 = 4(-1)(4)(-3)$

$\frac{1}{48} = \frac{48a}{48}$ $\frac{1}{48} = a$

2. $x^3+2x^2+4x+10 \div (x+2)$

$$\begin{array}{r|rrrr} & 1 & 2 & 4 & 10 \\ -2 & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 2 \end{array}$$

$$\boxed{x^2+4+\frac{2}{x+2}}$$

OUR 3 QUESTIONS

Anjali
Enrico
Simon

1) $P(x) = 3x^2+9x+10$
find the quotient and remainder when $p(x)$ is divided by $(x+4)$

$$\begin{array}{r|l} & 3x - 3 \\ \hline x+4 & 3x^2+9x+10 \\ & 3x^2+12x \\ \hline & \quad -3x+10 \\ & \quad -3x-12 \\ \hline & \qquad 22 \end{array}$$

$$\boxed{= 3x-3+\frac{22}{x+4}}$$

2) Find all possible cubic polynomials with zeroes

$\pm 6,\ x+i$

$(x+6)(x-6)(x-(x+i))$

$(x-6)(x+6)(x-x-1)$

$(x^2-36)(-1)$

$-x^2i+36i$

3) A polynomial is $P(x) = -2x^4+7x^3-cx^2+dx+e$. Yintercept is 45 the graph passes the x-axis at $(4,0)$ and $(-1,0)$ Find $c+d$ there are also only 2roots

Roots: 4 ± 1

$P(4)=0 \quad -2(4)^4+7(4)^3-c(4)^2+d(4)+45$

$$\frac{-16c+4d}{4} = \frac{-19}{4} \rightarrow -4c+d=-4.75$$

Chapter 6 Review Question Assessment

*highlighted is our questions, we ran out of time, the sub said it was ok

YeonJun Ko
Period 3

1) When $f(x)$ is divided by x^2-x+6, the quotient is x^2+2x+1 with remainder $ax+b$. When $f(x)$ is Divided by $(x+2)$, the remainder is -8 and when Divided $(x-3)$, the remainder is 2. Find $f(x)$.

Solution) $f(x)=(x^2-x+6)(x^2+2x+1)+ax+b$

$f(-2)=(4+2+6)(4-4+1)-2a+b,\ 12-2a+b,\ -2a+b=-12$

$f(3)=(9-3+6)(9+6+1)+3a+b,\ (12)(16)+3a+b,\ 3a+b=-192$

$$\begin{array}{rl} -2a+b&=-12\\ -\ \ 3a+b&=-192\\ \hline -5a&=180\\ a&=-36\\ b&=-84 \end{array}$$

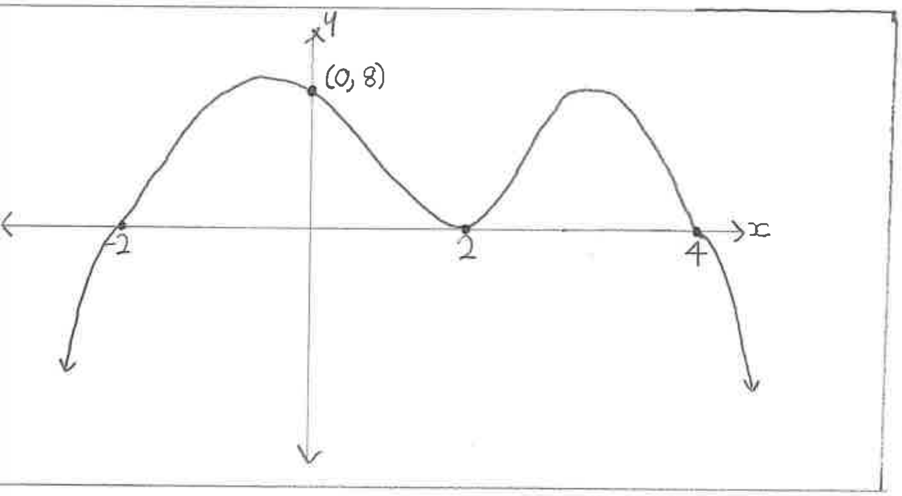
$f(x)=(x^4\underline{+2x^3}+\underline{x^2}\underline{-x^3}\underline{-2x^2}-x+\underline{6x^2}+12x+6)-36x-84$

$f(x)=x^4+x^3+5x^2+11x+6-36x-84$

$f(x)=x^4+x^3+5x^2-25x-78$

$\boxed{\text{Answer}=f(x)=x^4+x^3+5x^2-25x-78}$

2) Find the $f(x)$ equation of the graph.



Solution)

n=even, ↙ ↓

Roots = $x=-2$
$x=4$
$x=2$ (repeated)

⇓

$(x+2)(x-4)(x-2)^2=f(x)$

$f(x)=(x^2-2x-8)(x^2-4x+4)a$

$f(x)=(x^4\underline{-4x^3}+\underline{4x^2}\underline{-2x^3}+\underline{\cancel{8x^2}}-8x-\underline{\cancel{8x^2}}+32x-32)a$

$f(x)=(x^4-6x^3+4x^2+24x-32)a$

$8=0-0+0+0-32a$

$8=-32a$

$-\frac{1}{4}=a$

$\boxed{\text{Answer}=f(x)=-\frac{1}{4}(x^4-6x^3+4x^2+24x-32)}$

Circled is chosen Questions
-approved by sub teacher

Final Questions:

1. Write in the form $P(x) = Q(x)\,D(x) + R$

$$\frac{x^2 - 3x + 6}{x - 4}$$

$$x + 1 + \frac{10}{x-4}$$

$$\boxed{P(x) = (x+1)(x-4) + 10}$$

$x - 4 = 0$

$x = 4$

$$\begin{array}{r|rrr} 4 & 1 & -3 & 6 \\ & \downarrow & 4 & 4 \\ \hline & 1 & 1 & 10 \end{array}$$

2) $p(x) = x^4 + x^3 + ax^2 - x + b$. When $p(x)$ is divided by $(x+2)$, the remainder is -12. When $p(x)$ is divided by $(x-3)$, the remainder is 48. Find the Solutions of $p(x) = 0$.

Solution:

$x = -2 \rightarrow p(-2) = 16 - 8 + 4a + 2 + b = -12$

$4a + b = -22$ ①

$x = 3 \rightarrow p(3) = 81 + 27 + 9a - 3 + b = 48$

$9a + b = -57$ ②

② - ① $5a = -35 \Rightarrow a = -7$

$\Rightarrow b = 6$

$P(x) = x^4 + x^3 - 7x^2 - x + 6$

$\boxed{x = 1}$ $1 + 1 - 7 - 1 + 6 = 0 \rightarrow$

$$\begin{array}{r|rrrr|r} 1 & 1 & 1 & -7 & -1 & 6 \\ & & 1 & 2 & -5 & -6 \\ \hline & 1 & 2 & -5 & -6 & 0 \end{array}$$

$x^3 + 2x^2 - 5x - 6$

$\boxed{x = -1}$ $-1 + 2 + 5 - 6 = 0 \rightarrow$

$$\begin{array}{r|rrr|r} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$\rightarrow x^2 + x - 6$

$\Rightarrow = (x-2)(x+3) \Rightarrow \boxed{\begin{array}{l} x = 2 \\ x = -3 \end{array}}$

Cathy Wu

1) Given the polynomial $P(x) = x^3 - 2x^2 - ax + b$ and the roots $(x+3)$ and $(x-1)$, find a and b as well as the other roots.

$P(-3) = (-3)^3 - 2(-3)^2 - a(-3) + b = 0$

$-27 - 18 + 3a + b = 0$

$3a + b = 45$

$P(1) = (1)^3 - 2(1)^2 - a(1) + b = 0$

$1 - 2 - a + b = 0$

$-a + b = 1$

$3a + b = 45$

$- \quad -a + b = 1$

$4a = 44$

$\boxed{a = 11}$

$3(11) + b = 45$

$33 + b = 45$

$\boxed{b = 12}$

$P(x) = x^3 - 2x^2 - 11x + 12$

$(x+3)(x-1)$

$x^2 + 2x - 3$

$x - 4$

$x^2 + 2x - 3 \,\overline{)\, x^3 - 2x^2 - 11x + 12}$

$- \quad x^3 + 2x^2 - 3x$

$-4x^2 - 8x + 12$

$-4x^2 - 8x + 12$

0

$$\boxed{P(x) = (x+3)(x-1)(x-4)}$$

2) Given the polynomial $P(x) = x^3 - 5x^2 + 28x - 40$ and one of the roots is $2+4i$, find the remaining roots.

$(x - (2+4i))(x - (2-4i))$

$(x-2)^2 + 16$

$x^2 - 4x + 4 + 16$

$x^2 - 4x + 20$

$x - 2$

$x^2 - 4x + 20 \,\overline{)\, x^3 - 6x^2 + 28x - 40}$

$- \quad x^3 - 4x^2 + 20x$

$-2x^2 + 8x - 40$

$-2x^2 + 8x - 40$

0

$P(x) = (x - (2+4i))(x - (2-4i))(x - 2)$

Write in the form $P(x) = Q(x)\,D(x) + R$ Ronit Dasgupta

1. $\dfrac{x^2 - 3x + 6}{x - 4}$

$$\begin{array}{r|rrr} 4 & 1 & -3 & 6 \\ & \downarrow & 4 & 4 \\ \hline & 1x & 1 & 10 \end{array}$$

$$\boxed{P(x) = (x+1)(x-4) + 10}$$

$$\boxed{x + 1 + \frac{10}{x-4}} \rightarrow \boxed{(x+1)(x-4) + 10}$$

Find the sum and product of the roots of :

2. $2x^2 - 3x + 4 = 0$

sum: $\dfrac{-b}{a} \rightarrow \dfrac{-(-3)}{2} = \boxed{\dfrac{3}{2}}$

product: $\dfrac{c}{a} \rightarrow \dfrac{4}{2} = \boxed{2}$

3. Find the constant k and hence factorise the polynomial if :

$2x^3 + x^2 + kx - 4$ has the factor $(x+2)$

$2(-2)^3 + (-2)^2 + k(-2) - 4 = 0$

$16 + 4 - 2k - 4 = 0$

$-2k - 16 = 0$

$\dfrac{-2k}{-2} = \dfrac{16}{-2}$

$\boxed{k = -8}$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -8 & -4 \\ & \downarrow & -4 & 6 & 4 \\ \hline & 2x^2 & -3x & -2 & 0 \end{array} \; \nwarrow \text{remainder}$$

$2x^2 - 3x - 2 + \dfrac{0}{x+2}$

$$\boxed{P(x) = (x+2)(2x+1)(x-2)}$$

Andreea D, Joseph L, Jeslyn C, Puneet P

① Divide synthetically and find the remainder - Andreea

$(x^3+x^2+56) \div (x+1)$

$$\begin{array}{r|rrr} & 1 & 1 & 56 \\ -1 & & -1 & 0 \\ \hline & 1 & 0 & 56 \end{array}$$

$(x^2) + (56/_{x-1})$

② The polynomial $P(x) = x^3 + rx^2 + 5x - 9$ has one root that is equal to $(-2-\sqrt{5}i)$, find the other roots - Puneet

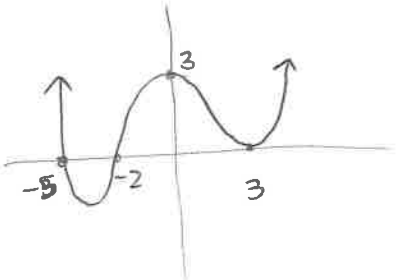
$(-2-\sqrt{5}i)(-2+\sqrt{5}i) \Rightarrow x^2+4x+9$

$(-2-\sqrt{5}i)^3 + (-2-\sqrt{5}i)^2 r + 5(-2-\sqrt{5}i) - 9 = 0$

$r=3$

$(x-1)(x+2+\sqrt{5}i)(x+2-\sqrt{5}i)(x-1)$

③ Find the equation from the graph



$3 = a(5)(2)(9)$

$3 = a(90)$

$a = \frac{1}{30}$

$P(x) = \frac{1}{30}(x+5)(x+2)(x-3)^2$

Final Questions

① $f(x) = 2x^3 + ax^2 - 22x - 24$

If $(2x+3)$ is a factor, find a where $a \in R$ and the other factors

$2\left(-\frac{3}{2}\right)^3 + a\left(-\frac{3}{2}\right)^2 - 22\left(-\frac{3}{2}\right) - 24 = 0$

$-\frac{24}{4} + \frac{9}{4}a + 33 - 24 = 0$

$-\frac{27}{4} + 9 + \frac{9}{4}a = 0$

$\frac{9}{4}a = \frac{-9}{4}a$

$a = -1$

$f(x) = 2x^3 - x^2 - 22x - 24$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & -22 & -24 \\ & \downarrow & -3 & 6 & 24 \\ \hline & 2 & -4 & -16 & 0 \end{array}$$

$(2x^2 - 4x - 16) \div 2$

$x^2 - 2x - 8$

$\boxed{(x-4)(x+2)(2x+3)}$

② When $x^3 + 2x^2 + ax + b$ is divided by $x-1$ the remainder is 4, and when divided by $x+2$ the remainder is 16. Find constants a and b

$x^3 + 2x^2 + ax + b$

$P(1) = 1 + 2 + a + b$

$a + b = 1$

$P(-2) = (-2)^3 + 2(-2)^2 + a(-2) + b$

$16 = -8 + 8 - 2a + b$

$16 = -2a + b$

$$\begin{array}{rl} & -2a + b = 16 \\ - & a + b = 1 \\ \hline & -3a = 15 \end{array} \Rightarrow \boxed{a = -5}$$

$-5 + b = 1$

$\boxed{b = 6}$

③ Perform the division $\frac{x^4 + 2x^2 - 1}{x+3}$ through the synthetic division method.
→ Write it in the form: $P(x) = Q(x)D(x) + R(x)$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 2 & 0 & -1 \\ & \downarrow & -3 & 9 & -33 & 99 \\ \hline & 1 & -3 & 11 & -33 & 98 \end{array}$$

$\boxed{(x^3 - 3x^2 + 11x - 33)(x+3) + 98}$

1) Given the polynomial $2x^4+7x^3-6x^2-17x+14$, find all roots.

Selected

$$\frac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 2} \Rightarrow 1, -1, 2, -2, 7, -7, 14, -14, \frac{1}{2}, \frac{-1}{2}, \frac{7}{2}, \frac{-7}{2}$$

$\Downarrow$ testing

$$f(x) = 2x^4+7x^3-6x^2-17x+14=0$$

$$\boxed{x = 1, -2, \frac{-7}{2}}$$

---

2) Find a 4th degree polynomial with zeros $\sqrt{7}$ and $5+i$.

$$(x+\sqrt{7})(x-\sqrt{7})(x-(5+i))(x-(5-i))$$

$$(x^2-7)(x^2-(5+i)x-(5-i)x+6)$$

$$(x^2-7)(x^2-10x+6)$$

$$\boxed{x^4-20x^3-x^2+70x-42}$$

Bryce Wang pd 3