HL Math Review Topic 4: Vectors

**Dot Product**

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| The **scalar dot product** of vectors  and  is | Find the angle  between vectors | Determine whether the angle is acute, right, or obtuse. |

**Cross Product**

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| The **vector cross product** of vectors  and  is | Tip: Use the determinant! | Remember  the signs! |

Properties of the Vector Cross Product

 is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ to both **a** and **b**. \_\_\_\_ for all **a**.

\_\_\_\_\_\_ for all **a** and **b**. Hence  and  have the \_\_\_\_\_\_\_\_ length and \_\_\_\_\_\_\_\_\_\_\_\_\_ direction.

 is called the **scalar triple product**. 

Length of :  = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡨 Use to find area of a parallelogram or a triangle.

**Equation of a Line**

Parametric Form Vector Form Cartesian Form

**Angle between lines**

Find the angle between the direction vectors and make it acute

**The distance from a point P to line m** given by .

Write down a generic point M on line m.

Find the value of  so that . (Geometric meaning?)

Find  for that value of .

 **Relationships between lines**  How to determine the relationship between lines in 3D

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| A single line is contained  in infinitely many planes. | Parallel lines  are coplanar. |
|  |  |
| Intersecting lines  are coplanar. | Skew lines do not intersect and are not parallel.  Skew lines are not coplanar. |

**Intersecting Lines**

Use parametric form – must use different parameters!

Solve the system formed by two pairs of equations.

Use the third pair to check. If it doesn’t check, then the lines are not intersecting (or you have an error).

Examples

Given: A(1, 2, -3), P(4, 5, 2), Line m: through A with direction vector , , 

1. Write the three forms of equations for line m.

2. Find the angle between L1 and L2.

3. Find the distance from P to line m.

4. Find the point of intersection of L1 and L2.

**Vector Equation of a Plane** Given point A in  and **b** and **c** are non-parallel vectors in .

**Cartesian Equation of a Plane** Given point A in  and  has normal vector **n**.

1. Find the equation of the plane containing A(1, 5, -2), B(-7, 12, 3), & C(4, -8, 9) in

a. Vector Form b. Cartesian Form

2. State the normal vector to the plane with equation 5x – 12y – 42z = 17.

**The angle between two planes**: Just like angle between two lines, but using the normal vectors.

**The angle between a line and a plane**: If the line direction vector and plane normal vector meet at angle  and the line direction vector and plane meet at angle , then .

3a. Find the parametric equations of the line through A(-1, 2, 3) and B(2, 0, -3).

b. Hence find where this line meets the plane with equation x – 2y + 3z = 26.

c. Find the angle between the line and the plane.

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| **Intersection of Three Planes** | * Use row reduction (or elimination) if you need to solve the 3x3 system. * Use the determinant to determine the existence of a unique solution. |
| If the planes then the intersection and there is (are)  Area all coincident is a plane infinitely many solutions  All meet in a common line is a line infinitely many solutions  All meet in a common point is a point one unique solution  Don’t meet like any of these DNE no solution |

**The Distance from a Point to a Plane**

Find the equation of the line through the point parallel to the normal vector.

Find the point where your line intersects the plane (foot of the normal from the point to the plane).

Find the distance between the original point and the foot of the normal.

4. Find the shortest distance from A(2, -1, 3) to the plane x – y + 2z = 27.