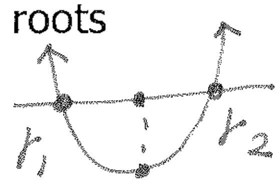
IB Math 1 1H Optimization

Three ways to find the vertex:

$y = a(x-h)^2 + k$

- Completing the Square (Vertex Form)
- Find the x-value of the vertex using $x = -\frac{b}{2a}$
- Find the x-value of the vertex using the midpoint of the roots



$x_1 = \frac{r_1 + r_2}{2}$

Example 1

Find the minimum y-value of $f(x) = 3x^2 - 24x + 63$

I $y = 3(x^2 - 8x + (4)^2) + 63 - (4)^2 \cdot 3$

$= 3(x-4)^2 + 15$

$\boxed{y \text{ min}: 15}$

$(4, 15)$

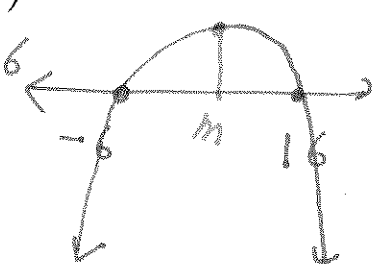
16
3
48

63
-48
5

Example 2

Find the maximum y-value of $g(x) = -8(x-16)(x+6)$.

$r_1 = 16 \quad r_2 = -6$



$m: \frac{16 + -6}{2} = \frac{10}{2} = 5.$

$g(5) = -8(5-16)(5+6)$

$= -8(-11)(11)$

$= 968$

$\boxed{y \text{ max}: 968}$

121
$\times 8$
968

IB Pre HL: Unit 1 F: Problem Solving with Quadratics **Notes: September 14**

Example1) A rectangle has length 3 cm longer than the width. Its area is 42 cm^2. Find the width.

W: x

L: $x+3$

A: $W \cdot L = x(x+3) = 42$

$x^2 + 3x = 42$

$x^2 + 3x - 42 = 0$

$x^2 + 3x = 42$

$+\left(\frac{3}{2}\right)^2 \quad +\left(\frac{3}{2}\right)^2$

$\left(x+\frac{3}{2}\right)^2 = 42 + \frac{9}{4}$

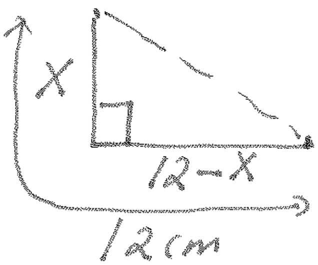
$\sqrt{\left(x+\frac{3}{2}\right)^2} = \sqrt{\frac{177}{4}}$

$x + \frac{3}{2} = \pm\sqrt{\frac{177}{4}}$

$x = -\frac{3}{2} \pm \sqrt{\frac{177}{4}}$

$= \boxed{\frac{-3+\sqrt{177}}{2}}$

Example 2) Is it possible to bend a 12 cm length of wire to form one leg of a right angled triangle with area 20 cm^2?



$A = \frac{x(12-x)}{2} = 20$

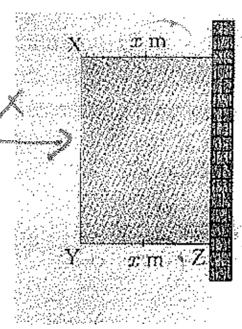
$x = ?$

Not possible

Example 3) A gardener has 40 m of fencing to enclose a rectangular garden plot, where one side is an existing brick wall. Suppose the two new equal sides are x m long.

a) Write an expression for the area enclosed by the fence in terms of x.

40−2x



$A = -2x(x-20)$

$A = x(40-2x) = -2x(x-20)$

b) Find the dimensions of the garden of maximum area.

$A = -2x(x-20)$

$r_1 = 0 \quad r_2 = 20$

$m = \frac{20}{2} = 10$

$A = -2(10)(10-20)$

$= \boxed{200 \text{ m}^2}$

$\boxed{10 \text{ m by } 20 \text{ m}}$