IB Pre HL More Practice for Polynomials Name: key

1. Factorize x^3+2x^2-x-2 into its linear factors. $= p(x)$

Step 1: List all possible roots. $\pm 1, \pm 2$

Step 2: Guess can check to find the first real root.

$$p(1) = 1+2-1-2=0 \Rightarrow \boxed{x=1}.$$

Step 3: Do synthetic division and/or factor and/or use Quadratic formula

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array} \Rightarrow x^2+3x+2 = (x+2)(x+1)$$

$\boxed{x=-1}$, $x=-2$ $\Leftarrow$ other roots

2. Factorize over rational numbers of

a) $4x^4-13x^2+9$.

$4x^2 \quad -9$

$x^2 \quad -1 \qquad -9-4=-13$.

$(4x^2-9)(x^2-1)$

$=\boxed{(2x+3)(2x-3)(x+1)(x-1)}$

b) x^4-2x^2-3

$(x^2-3)(x^2+1)$

$(x+\sqrt{3})(x-\sqrt{3})(x^2+1)$

3. When x^3+ax^2+bx+3 is divided by $x-2$ and $x-4$, the remainders are -3 and 15 respectively. Find a and b. $= f(x)$

$8+3$

$f(2)=-3 \Rightarrow 2^3+4a+2b+3=-3 \Rightarrow 4a+2b=-14$

$2a+b=-7$

Solve the system

$64+3=67$

$f(4)=15 \Rightarrow (4)^3+16a+4b+3=15 \Rightarrow \boxed{16a+4b=-52} \Rightarrow 4a+b=-13$

$2a=-6$

$a=-3$

$b=-1$

4. If $x-2$ is a factor of $p(x)=4x^3+kx-2$, find the value of k. Hence factorize p(x) fully.

$32-2=30$

$p(2)=4(2)^3+(2)(k)-2=0$

$2k=-30$ $\qquad$ $k=-15$

$p(x) = (x-2)(2x-2-\sqrt{3})(2x-2+\sqrt{3})$

5. When $p(x)=ax^3+bx^2+x-2$ is divided by, it has a remainder of 6. Given that $x+1$ is a factor of p(x), find all roots of p(x).

Even More Practice : No Calculators!!

1. Find a 3^{rd} degree polynomial with zeros $\frac{1}{2}$ and $2-i$. [3]

$P(x) = (2x-1)(x-(2-i))(x-(2+i))$

$= (2x-1)[(x-2)^2-(i)^2]$

$(2x-1)(x^2-4x+5)$

$= 2x^3-8x^2+10x$

$-x^2+4x-5$

$= 2x^3-9x^2+14x-5$

2. $(x-3)$ is a factor of $x^3+x^2-kx+15$. Find the value of k. [3]

$x=3$

$P(x)$

$27+9+15$

36
15

$P(3) = (3)^3+(3)^2-k(3)+15 = 0$

$-3k=51$ $k=\frac{51}{-3}=+17$

3. Find the quotient and remainder.

$\frac{3x^3-6x^2+4}{x^2+2x+3}$

$= 3x-9+\frac{9x+31}{x^2+2x+3}$

$3x-12 \Leftarrow Q(x)$

$x^2+2x+3 \overline{)\,3x^3-6x^2+0x+4}$

$-\ \underline{3x^3+6x^2+9x}$

$-9x^2-9x+4$

$-\ (-9x^2-18x-27)$

$15x+40 \Leftarrow R(x)$

4. Use synthetic division, and hence write the division in the form $P(x)=Q(x)D(x)+R(x)$.

$\frac{9x^4+3x^3+13x^2-2x-13}{x+2}$

$P(x) = 9x^4+3x^3+13x^2-2x-13.$

$= (9x^3-15x^2+43x-88)(x+2)+163$

-2	9	3	13	-2	-13
	↓	-18	30	-86	176
	$9x^3$	$-15x^2$	$43x$	-88	163

5. The equation $x^3+x^2+ax-4=0$ has one root equal to -2. Find the values of a and the remaining root.

$P(x) = x^3+x^2+ax-4$

$P(-2) = (-2)^3+(-2)^2-2a-4=0$

$\Rightarrow -8+4-2a-4=0$

$-2a=8$

$a=-4$

Remaining Root?

Roots: $-1, 2$.

6. The equation $2x^3+ax^2+bx+9=0$ has one repeated root equal to 3. Find the values of a and b and the remaining root.

Three roots: $x_1=3$, $x_2=3$, $x_3=C$

$2x^3+ax^2+bx+9=2(x-3)^2(x-x_3)$

$2(x^2-6x+9)(x-C)$

$=(2x^2-12x+18)(x-C)$

$=2x^3-2Cx^2-12x^2-12Cx$

$+18x-18C$

$$2x^3 + ax^2 + bx + 9 = 2x^3 + (-2c - 12)x^2 + (-12c + 18)x - 18c$$

$$9 = -18c \Rightarrow c = -\frac{1}{2}$$

$$a = (-2c - 12) \Rightarrow a = -11$$

$$b = (-12c + 18) \Rightarrow b = (-12)(-2) + 18$$

$$= 24 + 18$$

$$b = 12.$$